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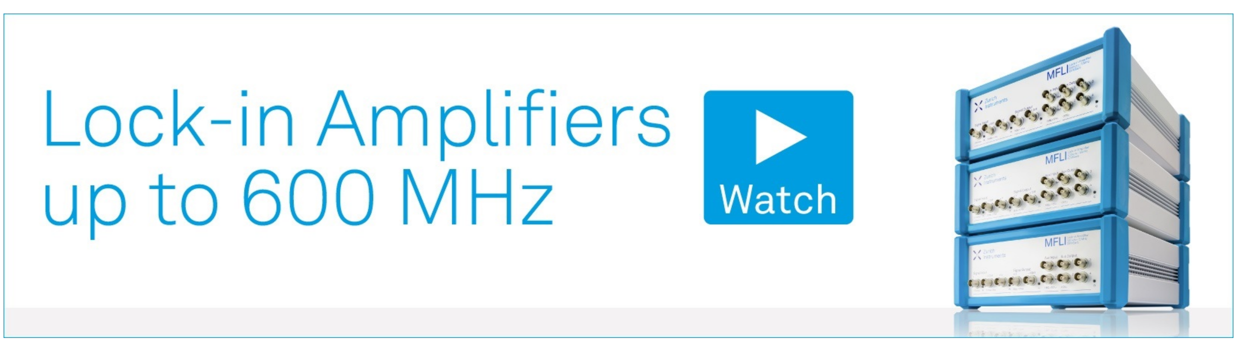

On Strong Pre-Continuity with Fuzzy Soft Sets

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Abstract. We adapt strong θ-precontinuity into fuzzy soft topology and investigate its properties. Also, the relations with the other types of continuities in fuzzy soft topological spaces are analized. Moreover, we give some new definitions.
Keywords: Fuzzy soft pre-$\theta-$open, fuzzy soft strong $\theta-$pre-continuity, fuzzy soft pre-$\theta-$closure and pre-$\theta-$interior points, fuzzy soft pre-regular and p-regular spaces, graph of a fuzzy soft function
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INTRODUCTION

Sometimes, we can not use traditional classical methods to handle some problems in some parts of real life such as medical sciences, social sciences, economics, engineering etc. Because, these problems involve various types of uncertainities. To cope with these problems, some new theories were given by scholars. Two of them are the theory of fuzzy sets and the theory of soft sets which were initiated by Zadeh [27] and Molodstov [16] in 1965 and 1999, respectively. These theories have always been used for dealing with these problems and constituted research areas for scientists to make investigations as in [3,4,8,9,10]. But, both of these theories have their inherent difficulties. Because of these difficulties, some new mathmatical tools were required. Then, Maji [15] presented the concept of fuzzy soft set in 2001 as a new mathmatical tool and investigated its properties such as De Morgan Law, the complement of a fuzzy soft set, fuzzy soft union, fuzzy soft intersection. By using the theory of fuzzy soft sets, the topological structures in geographic information systems (GIS) are analized in [10,11,12]. Also, some results on an application of fuzzy-soft-sets in decision making problem are presented by Roy and Maji in [23]. Ahmad and Kharal [2] made some additions to these properties and improved them. Tanay and Kandemir [26] investigated topological structures of fuzzy soft sets. Then, Roy and Samanta [24] introduced the definition of fuzzy soft topology over the initial universe in 2011.

Preliminaries

The fuzzy soft closure [20] of f_A, denoted by $Fcl(f_A)$, is the intersection of all fuzzy closed soft super sets of f_A. i.e.,

$$Fcl(f_A) = \sqcap\{h_D : h_D \ is \ fuzzy \ closed \ soft \ set \ and \ f_A \sqsubseteq h_D\}.$$

The fuzzy soft interior [20] of g_B, denoted by $Fint(g_B)$, is union of all fuzzy open soft subsets of g_B i.e.,

$$Fint(g_B) = \sqcup\{h_D : h_D \text{ is fuzzy open soft set and } h_D \sqsubseteq g_B \}.$$

Some scientists focused on these concepts and made researches as in [3, 10]

A fuzzy soft set f_A is said to be fuzzy soft preopen [1] (resp. fuzzy soft semiopen [10]) if $f_A \sqsubseteq Fint(Fcl(f_A))$ (resp. $f_A \sqsubseteq Fcl(Fint(f_A))$). The complement of a fuzzy soft preopen set is called fuzzy soft preclosed [1]. The fuzzy soft preclosure [1] of f_A, denoted by Fpcl(f_A), is the intersection of all fuzzy preclosed soft super sets of f_A i.e.,

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$$Fpcl(f_A) = \sqcap\{h_D : h_D \text{ is fuzzy preclosed soft set and } f_A \sqsubseteq h_D\}.$$

The fuzzy soft preinterior [1] of g_B, denoted by $Fpint(g_B)$, is union of all fuzzy open soft subsets of g_B i.e.,

$$Fpint(g_B) = \sqcup\{h_D : h_D \text{ is fuzzy preopen soft set and } h_D \sqsubseteq g_B\}.$$

The fuzzy soft set f_A in X_E is called fuzzy soft point [5] if $A = \{e\} \subseteq E$ and $f_A(e)$ is a fuzzy point in X i.e. there exists $x \in X$ such that $f_A(e)(x) = \alpha$ ($0< \alpha \leq 1$) and $f_A(e)(y) = 0$ for all $y \in X - \{x\}$. This fuzzy soft point will be denoted by e_x^α. Let f_A be a fuzzy soft set and e_x^α be a fuzzy soft point in X_E.

We say $e_x^\alpha \tilde{\in} f_A$ read as e_x^α belongs to f_A if $\alpha \leq f_A(e)(x)$. Let f_A and g_B be fuzzy soft sets in X_E. f_A is said to be soft quasi-coincident [4] with g_B , denoted by $f_A q g_B$, if there exist $e \in X$ and $x \in X$ such that $f_A(e)(x) + g_B(e)(x)>1$. If f_A is not quasi-coincident with g_B, then we write $f_A \bar{q} g_B$. A fuzzy soft point e_x^α of X_E is called a fuzzy soft θ-cluster point [14] of f_A if $Fcl(g_B)$ q f_A for every fuzzy soft open set g_B containing e_x^α.The union of all fuzzy soft θ-cluster points is of f_A is called fuzzy soft θ-closure [14] of f_A and denoted by $Fcl_\theta(f_A)$. A fuzzy soft set f_A is said to be fuzzy soft θ-closed [14] if $f_A = Fcl_\theta(f_A)$. The complement of a fuzzy soft θ-closed set is said to be fuzzy soft θ-open [14]. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two functions. Then, the pair (φ, ψ) is called a fuzzy soft mapping [5,11] from X_E to Y_K and denoted by $(\varphi, \psi) : X_E \to Y_K$. The image of each $f_A \in X_E$ under the fuzzy soft function (φ, ψ) will be denoted by $(\varphi, \psi)(f_A) = \varphi(f)_{\psi(A)}$ and the membership function of $\varphi(f)(\beta)$, for each β of $\psi(A)$, is defined as,

$$\varphi(f)(\beta) = \begin{cases} \bigvee_{x\in\Psi^{-1}(y)} \Big(\bigvee_{x\in\Psi^{-1}(\beta)\cap A} f_\alpha(x)\Big), & \Psi^{-1}(y) \neq \emptyset, \quad \Psi^{-1}(\beta) \cap A \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

for every $y \in Y$.

Definition 1 *A fuzzy soft point e_x^α of X_E is called a fuzzy soft pre-θ-cluster point of f_A if $Fpcl(g_B) q f_A$ for every fuzzy soft preopen set g_B containing e_x^α. The union of all fuzzy soft pre-θ-cluster points is of f_A is called fuzzy soft pre-θ-closure of f_A and denoted by $Fpcl_\theta(f_A)$. A fuzzy soft set f_A is said to be fuzzy soft pre-θ-closed if $f_A = Fpcl_\theta(f_A)$. The complement of a fuzzy soft pre-θ-closed set is said to be fuzzy soft pre-θ-open. In [21], fuzzy soft precontinuity is defined by A.Ponselvakumari and R.Selvi.*

We define fuzzy soft precontinuity in a different way as given:

Definition 2 *A fuzzy soft function $(\varphi, \psi) : X_E \to Y_K$ is said to be fuzzy soft precontinuous or fuzzy soft almost continuous (resp. fuzzy soft weakly precontinuous or fuzzy soft almost weakly continuous) if for each $e_x^\alpha$of X_E and each fuzzy soft open set g_B of Y_K containing $(\varphi, \psi)(e_x^\alpha)$, there exists a fuzzy soft preopen set f_A containing e_x^α such that $(\varphi, \psi)(f_A) \sqsubseteq g_B$ (resp. $(\varphi, \psi)(f_A) \sqsubseteq Fcl(g_B)$).*

Definition 3 *A fuzzy soft function $(\varphi, \psi) : X_E \to Y_K$ is said to be fuzzy soft strong θ-continuous (fuzzy soft strong θ- precontinuous) if for each e_x^α of X_E and each fuzzy soft open set g_B of Y_K containing $(\varphi, \psi)(e_x^\alpha)$, there exists a fuzzy soft preopen set f_A containing e_x^α such that*

$$(\varphi, \psi)(Fcl(fA)) \sqsubseteq g_B \ ((\varphi, \psi)(Fpcl(fA)) \sqsubseteq g_B).$$

Remark 1 *For a fuzzy soft function function $(\varphi, \psi) : X_E \to Y_K$:*

$$\begin{array}{c} Fuzzy\ Soft\ Strong\ \theta-\ continuity \\ \downarrow \\ Fuzzy\ Soft\ Strong\ \theta-\ pre-continuity \\ \downarrow \\ Fuzzy\ Soft\ Pre-continuity \end{array}$$

Theorem 1 *Let X_E and Y_K be fuzzy soft topological spaces. Then, the following properties are equivalent for a function $(\varphi, \psi) : X_E \to Y_K$;*

(1) (φ, ψ) is f.s.st.θ.p.c.;

(2) $(\varphi, \psi)^{-1}(g_B)$ is fuzzy soft pre-θ-open in X_E for every fuzzy soft open set g_B of Y_K;

(3) $(\varphi, \psi)^{-1}(g_B)$ is fuzzy soft pre-θ-closed in X_E for every fuzzy soft closed set g_B of Y_K;

(4) $(\varphi, \psi)(Fpcl_\theta(f_A)) \sqsubseteq Fcl((\varphi, \psi)(f_A))$ for every fuzzy soft subset f_A of X_E;

(5) $Fpcl_\theta((\varphi, \psi)^{-1}(g_B)) \sqsubseteq (\varphi, \psi)^{-1}(Fcl(g_B))$ for every fuzzy soft subset g_B of Y_K.

Conclusion

In this study, we have given the definition of strong θ-precontinuous function in fuzzy soft topology. We have focused on the properties of fuzzy soft strong θ-precontinuity in several types of fuzzy soft topological spaces and investigated the relationships with some other continuities which have been supported by a diagram and counter examples. This study is also an attempt to make a new approach to give a different definition for fuzzy soft graph function. Some valuable results that can be used in different disciplines are obtained and analized.

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