# DOI 10.1007/s12239–009–0051–z<br>FINITE ELEMENT ANALYSIS OF COLLAPSE OF FRONT SIDE RAILS WITH NEW TYPES OF CRUSH INITIATORS

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while at the same time meeting the demands of safety requirements. Reducing the cross-section of structural elements to achieve weight reduction can lead to adverse effects on passive safety of the vehicle. In such cases, necessary design modifications must be created to overcome the adverse effects. For this purpose, front rail columns with crush initiators are used in the front zone of cars. These shock-absorbing elements act as energy consuming devices that convert impact energy (kinetic energy) into plastic deformation energy. Simulation of this energy conversion phenomenon is the subject of this paper. The primary objective of this study is to computationally determine how various crush initiators can reduce the maximum crushing force and how different types of structural modifications affect the observed folding form. The ribs near the crash area are placed in two rows and four different configurations on all facing sides of the column in order to decrease reaction forces and absorb more kinetic energy. These structures are analyzed under axially loaded crushing forces using the explicit nonlinear finite element analysis solver ANSYS/LS-DYNA.

are designed with lighter weight to increase performance as the demands of safety requirements. Reducing the cross-<br>as o overcome the adverse effects on passive safety of the vehicle i. Note that is o concretent to advers ABSTRACT Today's vehicles are designed with lighter weight to increase performance and to how for the consumption of the increase performance and to increase the consumer fuel consumer and the consumer fuel consumer fuel **KEY WORDS** : Crush initiators, Crashworthiness, Energy absorption, Collapse analysis, FEA<br> **STRODUCTION**<br>
and Abramowicz introduce<br>
crushing behavior of a constrained to find infitiative space and dynamics (Weizbicki and 1. INTRODUCTION<br>The development of a light-weight vehicle is in great<br>demand for enhancement of fuel efficiency and dynamic<br>performance (Lee *et al.*, 2006). Front rail columns are the The development of a light-weight vehicle is in great demand for enhancement of fuel efficiency and dynamic performance (Lee *et al.*, 2006). Front rail columns are the chassis elements that absorb approximately 40% of the kinetic energy at the moment of a vehicle crash. If these columns are to rigid, this causes large negative chassis elements that absorb approximately 40% of the kinetic energy at the moment of a vehicle crash. If these columns are too rigid, this causes large negative accelerations and possible impact of the occupants with the windshield of the car because of delay in initial folding and persistent rigidity during a short fraction of time. Additionally, if the columns are too weak, they buckle too easily and objects impacted in the crash can enter the interior of the vehicle and injure the occupants. During a crash, a front rail column is expected to fold easily at the beginning and at the progressive stages of impact, becoming more rigid and folding like a bellows to convert the crash kinetic energy to plastic deformation energy. If the crush initiators are placed properly on the baseline column, then anticipated behavior can be obtained from them during the crash. Thin-walled structures are capable of carrying substantial loads with deflections and their energy absorption rate is high, as seen by their progressive folding ability. Progressive folding of a box column under axial crushing forces was studied experimentally by various researchers both theoretically and numerically. Wierzbicki

and Abramowicz introduced a theory that describes the crushing behavior of a class of thin-walled structures (Wierzbicki and Abramowicz, 1983).

Krauss and Laananen (1994) analyzed the effects of three crush initiators: a transverse bead on two sides along the cross-section of the tube, a diamond notch on each of the four corners, and a circular hole on the corners. They studied the geometry of peak and mean crushing forces on a thin-walled box column subjected to axial loading and compared the results with those obtained in the case of an unnotched baseline steel tube, showing a reduction of greater than 40% of initial peak crushing forces.

Cho *et al.* (2006) studied frontal crash optimizations of a frame with a nonuniform closed-hat section using le-type and dent-type crush initiators under an axial ding condition. They compared the results of design alysi front frame with a nonuniform closed-hat section using hole-type and dent-type crush initiators under an axial loading condition. They compared the results of design analysis of dent-type crush initiators with those of holetype crush initiators of the same size. DiPaolo *et al.* (2004) carried out a quasi-static experimental study on a welded AISI 304 stainless steel square box with collapse initiators, which consisted of a double set of mac carried out a quasi-static experimental study on a welded AISI 304 stainless steel square box with collapse initiators, which consisted of a double set of machined grooves along the transverse and full sidewall widths in opposite sidewalls, with a vertical offset for groove pairs in adjacent sidewalls, in order to investigate of the axial crush response of the component.

Cheng *et al.* (2006) conducted an experimental investition on square cross-sectioned AA6061-T6 aluminum<br>bes with through-hole discontinuities to observe their<br>their gation on square cross-sectioned AA6061-T6 aluminum tubes with through-hole discontinuities to observe their

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452 · crush characteristics and energy absorption capacities. -<br>n capacities. 2. MOTOR Kang and Huh (2000) derived a shell element formulation incorporating a contact algorithm, which adopts the plasticpredictor elastic corrector (PPEC) scheme during stress integration in order to keep track of the stress-strain relationship for rate-dependent models accurately. Their work numerically simulated the axial compression and folding of square tubes by elasto-plastic explicit finite element methods, and demonstrated the versatility of the algorithm. Nalepa (1990) showed an analytical crash simulation of an Opel Vectra during a 50 km/h frontal barrier impact using the explicit finite element method. These analyses provided valuable information about the front-end deformation behavior, speed variation, and energy absorption during a crash incident. Kim and Huh (2000) derived a limit analysis formulation with degenerate four-node shell elements for analysis of collapse behavior in square tubes and compared the results for load-carrying capacity and deformation modes with experimental values.

Vehicle structures are designed such that the crushing forces occurring in a crash are absorbed by the collapse behavior of the vehicle, thus reducing the forces transmitted to the occupant compartment. Geometric modifications called Crush Initiators create stress concentrations in a desired zone so that the structure material starts to locally plastify and absorb the energy, thus reducing forces on the occupant compartment. In fact, these shock-absorbing elements act as energy consuming devices that convert the impact energy (kinetic energy) into plastic deformation energy. It is exactly this energy conversion that is simulated. If a column deforms easily in the first and last stages of a crash event, this means the absorbed kinetic energy is large, and it can be concluded that the column safely deformed in this situation.

There are two major differences in this study that separate it from other work done in this field. First, stressconcentrated areas created by the axial forces affecting a baseline column are determined by using an FEA method. These areas are the locations where deformation generally starts. Secondly, new rib type crush initiators are proposed and are placed at locations where stress is concentrated. Moreover, different variations of the proposed rib type crush initiators are tested in order to find which variation dissipates more kinetic energy.

The aim of the study is to computationally determine how diverse crush initiators reduce the maximum crushing force and how different types of structural modifications change the folding form. First, an impact effect is created on a simple straight column. Second, columns with different types of crush initiators are analyzed under the same impact in order to determine load-carrying capacity, energy-absorbing ability, deflection type, and forcedeflection relationship. Third, the results belonging to the simple column are compared with those obtained with respect to the columns with various rib types of crush initiators.

## THEIR IMPORTANCE IN CRASHWORTHINESS

During the action of crushing forces, advancement of frontal deformation into the occupant compartment and occupant effects due to the action of inertial forces are the most dangerous situations. During deformation, progressive absorption of the kinetic energy and weakening of inertial effects reduces the occurrence of the abovementioned dangers. The energy-absorbing elements, which are present at the front section and are directly subject to the impact, are designed by taking absorption properties into account; they collapse in an accordion-like mode and reduce risk levels to a large extent. If they are designed as rigid, considering only the bending effect (which is possible under normal driving conditions), this can lead to increased risk levels (Xue et al., 1997).

The most important element of a car body, in regards to reducing both inertial effects and increasing energyabsorbing capability, are the chassis front side rails shown in Figure 1. Longitudinal rails belonging to the vehicle front frame, which is axially loaded (shown in Figure 1), also act as energy absorbers. These elements, starting from the windscreen line and reaching to the front bumper, behave like a truss element that reacts to the bending effects of the engine and suspension units fixed on them.

These front rails behave as an axially loaded column during a frontal impact event. These elements have disadvantages both in terms of initial folding force and energyabsorption abilities because they are rigidly designed to react to bending forces and axial torsional forces. Front rails, which react to impact forces by bending, can push the engine block into the occupant compartment. However, if these columns (rails) absorb the crushing energy by progressively folding until the moment they contact the engine, they can prevent this danger.

Properly designed front rails alone can absorb 40% of the impact energy (Krauss and Laananen, 1994) It is necessary that these structures fold up without bending deformation in order to increase the energy absorption ability. A force-deflection relationship for this kind of deformation is shown in Figure 2. Experimental study results are also given in Figure 2, as realized in the International



Figure 1. Front side rail (Courtesy of Mazda).



Figure 2. Crush force-deflection relationship of specimen (Kim and Huh, 2000).

Crash Analysis Center of George Washington University. Geometry, thickness, and material properties of the column and loading conditions used in this experiment are exactly simulated in the numerical analysis. The results obtained from the numerical analysis are compared with the experimental data. The force-deflection relationship is given in Figure 2 is a way to measure the success of idealization of the physical phenomena and options selected during crushing analysis. If the results obtained from the analysis are in good accordance with the test results, then it can be safely said that by changing the geometry and the model, results can be observed parametrically. It can be seen from Figure 2 that the force instantaneously increases until initial folding starts, and then decreases until the second folding starts.

The high intensity of the reaction force, which results from the action of the initial crushing force and negative acceleration, should be lowered by selected constructive modifications realized on the column. Then, serial foldings can occur around the mean crush force line. The area under the curves gives the deformation energy, and this is directly related to the impact-energy absorbing ability of the column. Constructive modifications made to reduce the initial folding force should not lead to reduction in this area. Similarly, the excellent bending strength of the columns should not be diminished, taking normal driving conditions into account. Therefore, many elements with different geometries should be analyzed in order to achieve optimization under the guidance of the parameter effect on the desired design.

In this study, the effects of different-sized crush initiators on the initial peak load and deformation of the column are analyzed with the ANSYS/LS-DYNA explicit analysis solver. This method should be used along with an FEA program which supports very high nonlinearity criteria

without giving rise to any problems because the explicit algorithm is used to calculate the absorbed kinetic energy, deformations, and peak and mean crushing forces of thinwalled tubes under the effect of axial crushing forces. An FEA program should exhibit certain features for use in such an analysis, such as:

- (1) A solution method which supports existing nonlinearity criteria without any convergence problems (Explicit Analysis),
- (2) Geometric Nonlinearity support, which in a stepwise manner considers the large changes in deflection that occur on the elements and changes in the angle of application of the load for inclusion in the calculation,
- (3) Consideration of material nonlinearity using rigidity values fit to the stress-strain curve in the next step by controlling the areas going beyond the yield point,
- (4) Ability to increase integration points inside an element against the case of elements which show structural instability, which causes a zero energy mode around the integration points (Hourglass Control),
- (5) A contact algorithm which considers the friction and force transfer of accordion-like folding surfaces among each other and the nodes which impact mass surface contacts of the mass (Automatic Single Surface),
- (6) A dynamic analysis which considers the inertial forces that arise from rigid body movements produced by very rapid deformation of each finite element (Rate-Dependent Plasticity),
- (7) An analysis method that supports stress wave propagation in an impact (Wave Propagation).

## 3. FINITE ELEMENT MODEL AND ANALYSIS **PARAMETERS**

## 3.1. Model Geometry and Material Properties

This study considers material properties similar to those described in the paper by Kang and Huh (2000). The nonlinear zone belonging to the material is defined as piecewise linear isotropic, a definition that the ANSYS/LS-



Figure 3. Piecewise linear approximation to stress–strain curve for the analyzed material.



Figure 4. Baseline column model.

DYNA program supports (see Figure 3). To simplify the calculations, it is assumed that the crushed mass is determined as rigid. In the study, various model configurations are parametrically compared with each other in order to reduce reaction forces.

The model column, its geometry, and the finite-element meshed version, as shown in Figure 4, was fixed at the bottom end, and a 400 kg mass stroke was applied from the top end at a specified velocity of 30 km/h in order to simulate an impact. The finite element mesh has 5052 fournode shell elements and 5047 nodal points, as shown in Figure 4. Numerical simulations are performed using the ANSYS/LS-DYNA (1998) solver.

Test results obtained at the National Crash Analysis Centre, George Washington University, by Zaouk et al. (1999) were compared with the computational analysis results of this study, with the geometry and the material properties the same. It can be observed that computational results show a significant correlation with the experimental data (Figure 2). In this study, although various model configurations are parametrically compared with each other in order to reduce reaction forces, the level of absorbed energy increases.

A square cross-sectioned thin-walled column is used to simulate the front side rails. The model column used in the analysis measures 80 mm  $\times$  80 mm, with a total length of 400 mm, and a wall thickness of 1.5 mm.

### 3.2. Loading Type and Deformation Relationship

There are initial imperfections applied to the square crosssectioned model column to start the initial crushing behavior. The column was subjected to free deformation related to stress wave propagation factors, which are selected according to the intensity and velocity of the applied load. A simple square cross-sectioned baseline column's progressive folding stages are shown in Figure 5.

The force-deflection relationship (which is of considerable importance for direct effect on negative inertia as a result of this deformation of square cross-sectioned baseline column) is illustrated in Figure 6.

As shown in Figure 6, the initial peak crushing load for the square cross-sectioned baseline column is very high, and this creates a negative situation for the occupants in the vehicle. As mentioned in certain references (see Krauss



 $0.002 s$  $0.030 s$ Figure 5. Progressive folding stages of a square crosssectioned baseline column (Eren, 2002).



Figure 6. Force deflection relationship of a square crosssectioned baseline column.

and Laananen (1994), DiPaolo et al. (2004), and Cheng et al. (2006)), it is possible to decrease the initial peak crushing load with construction modifications such as crosssectional beads, circular hole corner notches, and diamond notches on the corners of the square cross-sectioned column. The necessity for insertion of crush initiators at the corners can be seen from Figure 7, which indicates that corners are subjected to high axial stresses during the initial stages of the impact.

Axial rigidity of a baseline column can be decreased if the coordinates of the stress-concentrated areas are deter-



Figure 7. Stress distributions during the initial stages of the impact (Eren, 2002).

mined and if these areas are weakened with crush initiators. In this study, in order to locate the crush initiators explained in the subsequent section, a stress analysis is carried out to determine the stress-concentration areas during the initial stages of impact on the square cross-sectioned baseline column using the ANSYS/LS-DYNA finite element solver.

## 4.1. Crush Initiators

Crush initiators are the weakest points in the cross-section of the square cross-sectioned column, and they are deliberately placed on the column to initiate localized folding in order to reduce initial peak crushing load and to ensure a stable failure mode with significant energy absorbing ability (Krauss and Laananen, 1994).

Three different types of crush initiators were placed at the corners of a longitudinal column according to stress analysis results mentioned in the previous section (see Figure 8).

Differences, shown above, are created by constructive modifications, from the force-deflection relation perspective, as shown in Figure 9.

## 4.2 Convex and Concave Rib-Type Crush Initiators

Geometric imperfections applied at the corners of a square cross-sectioned column do not yield desirable effects because the corners of the column must endure forces acting from every direction during normal driving conditions and give strength to the column.

Therefore, in this study, convex and concave rib-type crush initiators are considered without any geometrical







Figure 9. Force-deflection relationships of column with different crush initiators.



(a) Rib-type crush initiators (b) Buckling (c) Stress distribution

Figure 10. Baseline column with rib-type crush initiators (Eren, 2002).

changes at the corners of the column. First, two convex ribtype crush initiators on opposite side walls and two concave rib-type crush initiators on adjacent sidewalls were formed on the square cross-sectioned column, as seen in Figure 10a. The location of the ribs was based on the stress analysis results mentioned in the previous section (Figure 7). Figure 10 shows a finite element model of the column with rib-type crush initiators. It can be seen from Figure 10 that rib-type crush initiators applied on the side walls of the column weakened the deformation areas and the collapse was initiated at the formed ribs. Figure 10 shows that stresses were concentrated at the rib area from the initial stages of the deformation. Different variations of the ribtype crush initiators under study are given in Figure 11 and Table 1.

The aim of these variations is to reduce peak crushing force at impact, and to increase the amount of energy dissipated during progressive folding, while making secondary and subsequent folding difficult.

A total of 4117 shell elements are used during the analysis of square-sectioned columns with type 1, 2, 3, and 4 crush initiators. The finite element type was type 163 (Explicit Thin Structural Shell). SHELL163 is a 4-node element with both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element



Figure 11. Four different variations of rib-type crush initiators and their crash analysis results (Eren, 2002).

456 · has 12 degrees of freedom at each node: translations, accelerations, and velocities in the nodal  $x$ ,  $y$ , and  $z$ directions and rotations about the nodal  $x$ ,  $y$ , and  $z$ -axes.

Load step options used for the analysis were:



In the analysis presented here, the magnitude of the hourglass control factor is varied according to the initial velocity in order to accurately reflect the strain rate dependency.

From the analysis of type 1, 2, 3, and 4 geometric imperfections applied to the sidewalls of a square cross-sectioned column, it is seen that the type 1 modification absorbed more crushing energy than the other types, as well as the baseline column. The absorbed crush energy value for the baseline column was  $E_k$ =3513 Joules at 10 cm deflection and the initial peak crash load was  $F_{\text{max}}$ =129.5 kN. These values for type 1 modification are  $E_k$ =4408 Joule and  $F_{max}$ =59.1 kN respectively.

In Figure 12, the crush initiator zone is zoomed in order to depict the details of the mesh. 1980 out of 4117 shell elements are used at the first 10 cm of the column to obtain more accurate results.

When circular-hole-type, diamond-notch-type, and beadtype crush initiators (see Figure 8) are considered, peak crash force is 121.05 kN, 118.5 kN, and 95.7 kN, and absorbed energy is 4073 Joules, 4037 Joules, and 3174 Joules, respectively. When these results are compared with those given in Table 2, it can clearly be seen that the column with rib-type crush initiators located on the sidewalls gives better results from a perspective of crashworthiness.

Initial peak crash loads and absorbed kinetic energy values for a column with rib-type crush initiators are given in Table 2.

Table 1. Details of type 1, 2, 3, and 4 geometric imperfections.

Imper- fection Type	Opposite side walls	Adjacent side walls	Row	Average mesh size (node to node dist.)
Type 1	convex type rib convex type rib		1	$2.21$ mm
	convex type rib convex type rib		$\mathfrak{D}_{\mathfrak{p}}$	
Type 2		concave type rib concave type rib	1	$2.19$ mm
	convex type rib convex type rib		$\mathfrak{D}$	
Type 3		convex type rib concave type rib	1	$2.17$ mm
	concave type rib convex type rib		$\mathcal{D}$	
Type 4	concave type rib convex type rib		1	$2.16$ mm
		convex type rib concave type rib	$\mathcal{D}$	



Figure 12. Mesh details at the crush initiator zone.

Table 2. Peak crash loads and absorbed energy values for type 1, 2, 3, and 4 imperfections.

	$F_{max}$ (kN)	EK(J)
Type 1 $(o)$	59.1	4408
Type 2 $(\blacksquare)$	58.5	4096
Type $3 (+)$	56.9	2934
Type 4 $(\triangle)$	57.7	3278



Figure 13 Force-deflection relationships belonging to type 1, 2, 3, and 4 bead crush initiators (Eren, 2002).

Force-deflection relationships with respect to type 1, 2, 3, and 4 rib-type crush initiators are depicted in Figure 13.

Figure 13 shows that the type 1 geometric imperfection has higher secondary and subsequent folding load values than the other configurations, which means that folding after the initial event requires more crushing load in order to fold and absorbs more crushing energy.

Computational nonlinear explicit analysis is employed to decrease the initial peak crushing load and to increase the crash energy absorption capacity of a square cross-sectioned column during an impact. Rib-type crush initiators are introduced to the column, and analysis of the collapse behavior is conducted on a column with rib-type crush initiators using the ANSYS/LS-DYNA explicit analysis

solver. The results obtained from the analysis of a square cross-sectioned baseline column were compared with those of type 1 crush initiators placed on the baseline column, and it seen that the initial peak crash load is reduced from 129.5 kN to 59.1 kN (see Figure 13 and Table 2). In the meantime, if the type 1 crush initiators were placed on the baseline column, the absorbed kinetic energy (deformation energy) value for the baseline column is increased from 3513 Joules at 10 cm deflection to 4408 Joules. When a square-sectioned column with circular-hole-type, diamondnotch-type, and bead-type crush initiators are considered, peak crash force is 121.05 kN, 118.5 kN, and 95.7 kN, and absorbed energy is 4073 Joules, 4037 Joules, and 3174 Joules at 10 cm, respectively (see Figure 9). When these results are compared with that of the columns with type 1, 2, 3, and 4 crush initiators, it can be clearly seen that the column with rib-type crush initiators located on the sidewalls gives better results from the perspective of reduction of crash force and increase in absorbed kinetic energy (see Figure 13 and Table 2). This means that forward acceleration of the occupants at the crash moment is diminished and the risk to the other car involved in the crash is also lowered.

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