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MATHEMATICS

# Generalized Sasakian space forms with semi-symmetric non-metric connections

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**Abstract.** We introduce generalized Sasakian space forms with semi-symmetric non-metric connections. We show the existence of a generalized Sasakian space form with a semi-symmetric non-metric connection and give some examples by warped products endowed with semi-symmetric non-metric connections.

**Key words:** generalized Sasakian space form, warped product, semi-symmetric non-metric connection.

## 1. INTRODUCTION

A semi-symmetric linear connection in a differentiable manifold was introduced by Friedmann and Schouten in [5]. Hayden [6] introduced the idea of a metric connection with torsion in a Riemannian manifold. In [15], Yano studied a semi-symmetric metric connection in a Riemannian manifold. In [1], Agashe and Chafle introduced the notion of a semi-symmetric non-metric connection and studied some of its properties.

Furthermore, in [2], Alegre, Blair, and Carriazo introduced the notion of a generalized Sasakian space form and gave many examples of these manifolds by using some different geometric techniques.

In [11], the present authors studied a warped product manifold endowed with a semi-symmetric metric connection and found relations between curvature tensors, Ricci tensors, and scalar curvatures of the warped product manifold with this connection. Moreover, in [12], we considered generalized Sasakian space forms with semi-symmetric metric connections.

Motivated by the above studies, in the present paper, we consider generalized Sasakian space forms admitting semi-symmetric non-metric connections. We obtain the existence theorem of a generalized Sasakian space form with a semi-symmetric non-metric connection and give some examples by the use of warped products.

The paper is organized as follows: In Section 2, we give a brief introduction to the semi-symmetric non-metric connection. In Section 3, the definition of a generalized Sasakian space form is given and we introduce generalized Sasakian space forms endowed with semi-symmetric non-metric connections. In the last section, the existence theorem of a generalized Sasakian space form with a semi-symmetric non-metric connection is given by warped product $\mathbb{R}\times_f N$, where $N$ is a generalized complex space form. In that section we obtain some examples of generalized Sasakian space forms with non-constant functions with respect to semi-symmetric non-metric connections.

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## 2. SEMI-SYMMETRIC NON-METRIC CONNECTION

Let $M$ be an $n$-dimensional Riemannian manifold with Riemannian metric $g$. If $\nabla$ is the Levi-Civita connection of a Riemannian manifold $M$, a linear connection $\overset{\circ}{\nabla}$ is given by

$$\overset{\circ}{\nabla}_X Y = \nabla_X Y + \eta(Y)X, \tag{1}$$

where $\eta$ is a 1-form associated with the vector field $\xi$ on $M$ defined by

$$\eta(X) = g(X, \xi), \tag{2}$$

(see [1]). By the use of (1), the torsion tensor $T$ of the connection $\overset{\circ}{\nabla}$

$$T(X,Y) = \overset{\circ}{\nabla}_X Y - \overset{\circ}{\nabla}_Y X - [X,Y] \tag{3}$$

satisfies

$$T(X,Y) = \eta(Y)X - \eta(X)Y. \tag{4}$$

A linear connection $\overset{\circ}{\nabla}$ satisfying (4) is called a *semi-symmetric connection*. $\overset{\circ}{\nabla}$ is called a *metric connection* if

$$\overset{\circ}{\nabla} g = 0.$$

If $\overset{\circ}{\nabla} g \neq 0$, then $\overset{\circ}{\nabla}$ is said to be a *non-metric connection*. In view of (1), it is easy to see that

$$(\overset{\circ}{\nabla}_X g)(Y,Z) = -\eta(Y)g(X,Z) - \eta(Z)g(X,Y) \tag{5}$$

for all vector fields $X,Y,Z$ on $M$.

Therefore, in view of (4) and (5), $\overset{\circ}{\nabla}$ is a semi-symmetric non-metric connection.

Let $R$ and $\overset{\circ}{R}$ be curvature tensors of $\nabla$ and $\overset{\circ}{\nabla}$ of a Riemannian manifold $M$, respectively. Then $R$ and $\overset{\circ}{R}$ are related by

$$\overset{\circ}{R}(X,Y)Z = R(X,Y)Z - \alpha(Y,Z)X + \alpha(X,Z)Y \tag{6}$$

for all vector fields $X,Y,Z$ on $M$, where $\alpha$ is a $(0,2)$-tensor field denoted by

$$\alpha(X,Y) = (\nabla_X \eta)Y - \eta(X)\eta(Y),$$

(see [15]).

## 3. GENERALIZED SASAKIAN SPACE FORMS

Let $M$ be an $n$-dimensional almost contact metric manifold with an almost contact metric structure $(\varphi, \xi, \eta, g)$ consisting of a $(1,1)$ tensor field $\varphi$, a vector field $\xi$, a 1-form $\eta$, and a Riemannian metric $g$ on $M$ satisfying

$$\varphi^2 X = -X + \eta(X)\xi, \qquad \eta(\xi) = 1, \qquad g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y)$$

for all vector fields $X,Y$ on $M$ [4].

An almost contact metric structure of $M$ is said to be *normal* if $[\varphi,\varphi](X,Y) = -2d\eta(X,Y)\xi$, for any vector fields $X,Y$ on $M$, where $[\varphi,\varphi]$ denotes the Nijenhuis torsion of $\varphi$, given by $[\varphi,\varphi](X,Y) = \varphi^2[X,Y] + [\varphi X, \varphi Y] - \varphi[\varphi X, Y] - \varphi[X, \varphi Y]$. A normal contact metric manifold is called a *Sasakian manifold* [4].

It is well known that an almost contact metric manifold is Sasakian if and only if $(\nabla_X\varphi)Y = g(X,Y)\xi - \eta(Y)X$. Moreover, the curvature tensor $R$ of a Sasakian manifold satisfies $R(X,Y)\xi = \eta(Y)X - \eta(X)Y$. An almost contact metric manifold $M$ is a *trans-Sasakian manifold* [9] if there exist two functions $\alpha$ and $\beta$ on $M$ such that

$$(\nabla_X\varphi)Y = \alpha[g(X,Y)\xi - \eta(Y)X] + \beta[g(\varphi X,Y)\xi - \eta(Y)\varphi X] \tag{7}$$

for any vector fields $X,Y$ on $M$. From (7) it follows that

$$\nabla_X\xi = -\alpha\varphi X + \beta[X - \eta(X)\xi]. \tag{8}$$

If $\beta = 0$ (resp. $\alpha = 0$), then $M$ is said to be an *$\alpha$-Sasakian manifold* (resp. *$\beta$-Kenmotsu manifold*). Sasakian manifolds (resp. Kenmotsu manifolds [7]) appear as examples of $\alpha$-Sasakian manifolds ($\beta$-Kenmotsu manifolds), with $\alpha = 1$ (resp. $\beta = 1$).

Another kind of trans-Sasakian manifolds is that of *cosymplectic manifolds* [3], obtained for $\alpha = \beta = 0$. From (8), for a cosymplectic manifold it follows that

$$\nabla_X\xi = 0.$$

For an almost contact metric manifold $M$, a *$\varphi$-section* of $M$ at $p \in M$ is a section $\pi \subseteq T_pM$ spanned by a unit vector $X_p$ orthogonal to $\xi_p$ and $\varphi X_p$. The *$\varphi$-sectional curvature* of $\pi$ is defined by $K(X \wedge \varphi X) = R(X,\varphi X,\varphi X,X)$. A Sasakian manifold with constant $\varphi$-sectional curvature $c$ is called a *Sasakian space form*. Similarly, a Kenmotsu manifold with constant $\varphi$-sectional curvature $c$ is called a *Kenmotsu space form*. A cosymplectic manifold with constant $\varphi$-sectional curvature $c$ is called a *cosymplectic space form*.

Given an almost contact metric manifold $M$ with an almost contact metric structure $(\varphi,\xi,\eta,g)$, $M$ is called a *generalized Sasakian space form* if there exist three functions $f_1, f_2$, and $f_3$ on $M$ such that

$$\begin{aligned} R(X,Y)Z =& f_1\{g(Y,Z)X - g(X,Z)Y\} + f_2\{g(X,\varphi Z)\varphi Y - g(Y,\varphi Z)\varphi X + 2g(X,\varphi Y)\varphi Z)\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \end{aligned} \tag{9}$$

for any vector fields $X,Y,Z$ on $M$, where $R$ denotes the curvature tensor of $M$. If $f_1 = \frac{c+3}{4}$, $f_2 = f_3 = \frac{c-1}{4}$, then $M$ is a Sasakian space form; if $f_1 = \frac{c-3}{4}$, $f_2 = f_3 = \frac{c+1}{4}$, then $M$ is a Kenmotsu space form; if $f_1 = f_2 = f_3 = \frac{c}{4}$, then $M$ is a cosymplectic space form.

Let $\overset{\circ}{\nabla}$ be semi-symmetric non-metric connection on an almost contact metric manifold $M$. We define $M$ as a *generalized Sasakian space form with semi-symmetric non-metric connection* if there exist four functions $\widetilde{f}_1, \widetilde{f}_2, \widetilde{f}_3$, and $\widetilde{f}_4$ on $M$ such that

$$\begin{aligned} \overset{\circ}{R}(X,Y)Z =& \widetilde{f}_1\{g(Y,Z)X - g(X,Z)Y\} + \widetilde{f}_2\{g(X,\varphi Z)\varphi Y - g(Y,\varphi Z)\varphi X + 2g(X,\varphi Y)\varphi Z\} \\ &+ \widetilde{f}_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X\} + \widetilde{f}_4\{g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\} \end{aligned}$$

for any vector fields $X,Y,Z$ on $M$, where $\overset{\circ}{R}$ denotes the curvature tensor of $M$ with respect to semi-symmetric non-metric connection $\overset{\circ}{\nabla}$.

**Example 3.1.** A cosymplectic space form with a semi-symmetric non-metric connection is a generalized Sasakian space form with a semi-symmetric non-metric connection such that $\widetilde{f}_1 = \widetilde{f}_2 = \widetilde{f}_4 = \frac{c}{4}$ and $\widetilde{f}_3 = \frac{c-4}{4}$.

**Example 3.2.** A Kenmotsu space form with a semi-symmetric non-metric connection is a generalized Sasakian space form with a semi-symmetric non-metric connection such that $\widetilde{f}_1 = \widetilde{f}_3 = \frac{c-7}{4}$ and $\widetilde{f}_2 = \widetilde{f}_4 = \frac{c+1}{4}$.

**Remark 3.3.** A Sasakian space form with a semi-symmetric non-metric connection is not a generalized Sasakian space form with a semi-symmetric non-metric connection.

If $(M,J,g)$ is a Kaehlerian manifold (i.e., a smooth manifold with a $(1,1)$-tensor field $J$ and a Riemannian metric $g$ such that $J^2=-I$, $g(JX,JY)=g(X,Y)$, $\nabla J=0$ for arbitrary vector fields $X,Y$ on $M$, where $I$ is identity tensor field and $\nabla$ the Riemannian connection of $g$) with constant holomorphic sectional curvature $K(X\wedge JX)=c$, then it is said to be a *complex space form* if its curvature tensor is given by

$$R(X,Y)Z=\frac{c}{4}\{g(Y,Z)X-g(X,Z)Y+g(X,JZ)JY-g(Y,JZ)JY+2g(X,JY)JZ\}.$$

Models for these spaces are $\mathbb{C}^n$, $\mathbb{C}P^n$, and $\mathbb{C}H^n$, depending on $c=0$, $c>0$, or $c<0$.

More generally, if the curvature tensor of an almost Hermitian manifold $M$ satisfies

$$R(X,Y)Z=F_1\{g(Y,Z)X-g(X,Z)Y\}+F_2\{g(X,JZ)JY-g(Y,JZ)JY+2g(X,JY)JZ\},$$

where $F_1$ and $F_2$ are differentiable functions on $M$, then $M$ is said to be a *generalized complex space form* (see [13] and [14]).

## 4. EXISTENCE OF A GENERALIZED SASAKIAN SPACE FORM WITH A SEMI-SYMMETRIC NON-METRIC CONNECTION

Let $(M_1,g_{M_1})$ and $(M_2,g_{M_2})$ be two Riemannian manifolds and $f$ a positive differentiable function on $M_1$. Consider the product manifold $M_1\times M_2$ with its projections $\pi:M_1\times M_2\to M_1$ and $\sigma:M_1\times M_2\to M_2$. The *warped product* $M_1\times_f M_2$ is the manifold $M_1\times M_2$ with the Riemannian structure such that

$$\|X\|^2=\|\pi_*(X)\|^2+f^2(\pi(p))\,\|\sigma_*(X)\|^2$$

for any tangent vector $X\in TM$. Thus we have that

$$g=g_{M_1}+f^2g_{M_2} \tag{10}$$

holds on $M$. The function $f$ is called the *warping function* of the warped product [8].

We need the following lemma from [10] for later use:

**Lemma 4.1.** *Let $M=M_1\times_f M_2$ be a warped product and $R$ and $\overset{\circ}{R}$ denote the Riemannian curvature tensors of $M$ with respect to the Levi-Civita connection and the semi-symmetric non-metric connection, respectively. If $X,Y,Z\in\chi(M_1)$, $U,V,W\in\chi(M_2)$ and $\xi\in\chi(M_1)$, then*

(i) $\overset{\circ}{R}(X,Y)Z\in\chi(M_1)$ *is the lift of* ${}^{M_1}\overset{\circ}{R}(X,Y)Z$ *on* $M_1$,

(ii) $\overset{\circ}{R}(V,X)Y=[-H^f(X,Y)/f-g(Y,\nabla_X\xi)+\eta(X)\eta(Y)]V$,

(iii) $\overset{\circ}{R}(X,Y)V=0$,

(iv) $\overset{\circ}{R}(V,W)X=0$,

(v) $\overset{\circ}{R}(X,V)W=-g(V,W)[(\nabla_X\,\mathrm{grad}f)/f+(\xi f/f)X]$,

(vi) $\overset{\circ}{R}(U,V)W={}^{M_2}R(U,V)W-\{\|\mathrm{grad}f\|^2/f^2+(\xi f/f)\}[g(V,W)U-g(U,W)V]$.

Now, let us begin with the existence theorem of a generalized Sasakian space form with a semi-symmetric non-metric connection:

**Theorem 4.2.** *Let $N(F_1,F_2)$ be a generalized complex space form. Then the warped product $M=\mathbb{R}\times_f N$ endowed with the almost contact metric structure $(\varphi,\xi,\eta,g)$ with a semi-symmetric non-metric connection is a generalized Sasakian space form with a semi-symmetric non-metric connection such that*

$$\widetilde{f_1}=\tfrac{(F_1\circ\pi)}{f^2}-\left[\left(\tfrac{f'}{f}\right)^2+\tfrac{f'}{f}\right],\quad \widetilde{f_2}=\tfrac{(F_2\circ\pi)}{f^2},$$

$$\widetilde{f_3}=\tfrac{(F_1\circ\pi)}{f^2}-\left[\left(\tfrac{f'}{f}\right)^2+\tfrac{f'}{f}\right]+\tfrac{(f''-f)}{f},\quad \widetilde{f_4}=\tfrac{(F_1\circ\pi)}{f^2}-\left[\left(\tfrac{f'}{f}\right)^2-\tfrac{f''}{f}\right].$$

*Proof.* For any vector fields $X,Y,Z$ on $M$, we can write

$$X=\eta(X)\xi+U,$$

$$Y=\eta(Y)\xi+V,$$

and

$$Z=\eta(Z)\xi+W,$$

where $U,V,W$ are vector fields on a generalized complex space form $N$. Since the structure vector field $\xi$ is on $\mathbb{R}$, then in view of Lemma 4.1 we have

$$\begin{aligned}\overset{\circ}{R}(X,Y)Z=&\eta(X)\eta(Z)\left[\frac{H^f(\xi,\xi)}{f}-1\right]V-\eta(X)g(V,W)[(\nabla_\xi\,\mathrm{grad}f)/f+(\xi f/f)\xi]\\&-\eta(Y)\eta(Z)\left[\frac{H^f(\xi,\xi)}{f}-1\right]U+\eta(Y)g(U,W)[(\nabla_\xi\,\mathrm{grad}f)/f+(\xi f/f)\xi]\\&+{}^N R(U,V)W-\{\|\mathrm{grad}f\|^2/f^2+(\xi f/f)\}[g(V,W)U-g(U,W)V].\end{aligned}\tag{11}$$

Since $f=f(t)$, $\mathrm{grad}f=f'\xi$, we get

$$\nabla_\xi\,\mathrm{grad}f=f''\xi+f'\nabla_\xi\xi.$$

By virtue of Proposition 35 on page 206 in [8], since $\nabla_\xi\xi=0$, the above equation reduces to

$$\nabla_\xi\,\mathrm{grad}f=f''\xi.\tag{12}$$

Moreover, we have

$$H^f(\xi,\xi)=g(\nabla_\xi\,\mathrm{grad}f,\xi)=f'',\tag{13}$$

$$\|\mathrm{grad}f\|^2=(f')^2,\quad \xi f=g(\mathrm{grad}f,\xi)=f'.\tag{14}$$

By virtue of equations (10), (12), (13), and (14) in (11) and by using the fact that $N$ is a generalized complex space form, we have

$$\begin{aligned}\overset{\circ}{R}(X,Y)Z=&\left(\frac{f''-f}{f}\right)\{\eta(X)\eta(Z)V-\eta(Y)\eta(Z)U\}\\&+\left(\frac{f''+f'}{f}\right)\{f^2g_{M_2}(U,W)\eta(Y)\xi-f^2g_{M_2}(V,W)\eta(X)\xi\}\\&+(F_1\circ\pi)\{g_{M_2}(V,W)U-g_{M_2}(U,W)V\}\\&+(F_2\circ\pi)\{g_{M_2}(U,JW)JV-g_{M_2}(V,JW)JU+2g_{M_2}(U,JV)JW\}\\&+\left(\left(\frac{f'}{f}\right)^2+\frac{f'}{f}\right)\{f^2g_{M_2}(U,W)V-f^2g_{M_2}(V,W)U\}.\end{aligned}$$

In view of Equation (10) and by the use of the relations between the vector fields $X,Y,Z$ and $U,V,W$, the above equation reduces to

$$\begin{aligned}\overset{\circ}{R}(X,Y)Z = &\left(\frac{(F_1\circ\pi)}{f^2} - \left[\left(\frac{f'}{f}\right)^2 + \frac{f'}{f}\right]\right)\{g(Y,Z)X - g(X,Z)Y\} \\ &+ \left(\frac{F_2\circ\pi}{f^2}\right)\{g(X,\varphi Z)\varphi Y - g(Y,\varphi Z)\varphi X + 2g(X,\varphi Y)\varphi Z\} \\ &+ \left(\frac{(F_1\circ\pi)}{f^2} - \left[\left(\frac{f'}{f}\right)^2 + \frac{f'}{f}\right] + \frac{(f''-f)}{f}\right)\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X\} \\ &+ \left(\frac{(F_1\circ\pi)}{f^2} - \left[\left(\frac{f'}{f}\right)^2 - \frac{f''}{f}\right]\right)\{g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi\}.\end{aligned}$$

Therefore, we complete the proof of the theorem. □

So we can state the following corollaries:

**Corollary 4.3.** *If $N(a,b)$ is a generalized complex space form with constant functions, then we have a generalized Sasakian space form with a semi-symmetric non-metric connection with non-constant functions such that*

$$\widetilde{f_1} = \tfrac{a}{f^2} - \left[\left(\tfrac{f'}{f}\right)^2 + \tfrac{f'}{f}\right], \quad \widetilde{f_2} = \tfrac{b}{f^2},$$

$$\widetilde{f_3} = \tfrac{a}{f^2} - \left[\left(\tfrac{f'}{f}\right)^2 + \tfrac{f'}{f}\right] + \tfrac{(f''-f)}{f}, \quad \widetilde{f_4} = \tfrac{a}{f^2} - \left[\left(\tfrac{f'}{f}\right)^2 - \tfrac{f''}{f}\right].$$

**Corollary 4.4.** *If $N(c)$ is a complex space form, we have*

$$\widetilde{f_1} = \tfrac{c}{4f^2} - \left[\left(\tfrac{f'}{f}\right)^2 + \tfrac{f'}{f}\right], \quad \widetilde{f_2} = \tfrac{c}{4f^2},$$

$$\widetilde{f_3} = \tfrac{c}{4f^2} - \left[\left(\tfrac{f'}{f}\right)^2 + \tfrac{f'}{f}\right] + \tfrac{(f''-f)}{f}, \quad \widetilde{f_4} = \tfrac{c}{4f^2} - \left[\left(\tfrac{f'}{f}\right)^2 - \tfrac{f''}{f}\right].$$

*Hence, the warped product $M = \mathbb{R}\times_f N(c)$ is a generalized Sasakian space form with a semi-symmetric non-metric connection $\overset{\circ}{\nabla}$.*

Thus, for example, the warped product $\mathbb{R}\times_f \mathbb{C}^n$ with non-constant functions

$$\widetilde{f_1} = -\left[\left(\tfrac{f'}{f}\right)^2 + \tfrac{f'}{f}\right], \quad \widetilde{f_2} = 0,$$

$$\widetilde{f_3} = -\left[\left(\tfrac{f'}{f}\right)^2 + \tfrac{f'}{f}\right] + \tfrac{(f''-f)}{f}, \quad \widetilde{f_4} = -\left[\left(\tfrac{f'}{f}\right)^2 - \tfrac{f''}{f}\right],$$

the warped product $\mathbb{R}\times_f \mathbb{C}P^n(4)$ with non-constant functions

$$\widetilde{f_1} = \tfrac{1}{f^2} - \left[\left(\tfrac{f'}{f}\right)^2 + \tfrac{f'}{f}\right], \quad \widetilde{f_2} = \tfrac{1}{f^2},$$

$$\widetilde{f}_3 = \frac{1}{f^2} - \left[\left(\frac{f'}{f}\right)^2 + \frac{f'}{f}\right] + \frac{(f''-f)}{f}, \quad \widetilde{f}_4 = \frac{1}{f^2} - \left[\left(\frac{f'}{f}\right)^2 - \frac{f''}{f}\right],$$

and the warped product $\mathbb{R} \times_f \mathbb{C}H^n(-4)$ with non-constant functions

$$\widetilde{f}_1 = -\frac{1}{f^2} - \left[\left(\frac{f'}{f}\right)^2 + \frac{f'}{f}\right], \quad \widetilde{f}_2 = -\frac{1}{f^2},$$

$$\widetilde{f}_3 = -\frac{1}{f^2} - \left[\left(\frac{f'}{f}\right)^2 + \frac{f'}{f}\right] + \frac{(f''-f)}{f}, \quad \widetilde{f}_4 = -\frac{1}{f^2} - \left[\left(\frac{f'}{f}\right)^2 - \frac{f''}{f}\right]$$

are generalized Sasakian space forms with semi-symmetric non-metric connections, respectively.

Hence, this method gives us some examples of generalized Sasakian space forms with semi-symmetric non-metric connections with arbitrary dimensions and non-constant functions.

## 5. CONCLUSION

Generalized Sasakian space forms with semi-symmetric non-metric connections are introduced. It is shown that if $N(F_1, F_2)$ is a generalized complex space form, then the warped product $M = \mathbb{R} \times_f N$ endowed with the almost contact metric structure $(\varphi, \xi, \eta, g)$ with a semi-symmetric non-metric connection is a generalized Sasakian space form with a semi-symmetric non-metric connection. Using this method, we obtain some examples of generalized Sasakian space forms with semi-symmetric non-metric connections with arbitrary dimensions and non-constant functions.

## REFERENCES

1. Agashe, N. S. and Chafle, M. R. A semi-symmetric non-metric connection on a Riemannian manifold. *Indian J. Pure Appl. Math.*, 1992, **23**(6), 399–409.
2. Alegre, P., Blair, D. E., and Carriazo, A. Generalized Sasakian-space-forms. *Israel J. Math.*, 2004, **141**, 157–183.
3. Blair, D. E. The theory of quasi-Sasakian structures. *J. Differ. Geom.*, 1967, **1**, 331–345.
4. Blair, D. E. *Riemannian Geometry of Contact and Symplectic Manifolds*. Birkhauser, Boston, 2002.
5. Friedmann, A. and Schouten, J. A. Über die Geometrie der halbsymmetrischen Übertragungen. *(German) Math. Z.*, 1924, **21**(1), 211–223.
6. Hayden, H. A. Subspace of a space with torsion. *P. Lond. Math. Soc. II Ser.*, 1932, **34**, 27–50.
7. Kenmotsu, K. A class of almost contact Riemannian manifolds. *Tôhoku Math. J.*, 1972, **24**, 93–103.
8. O'Neill, B. *Semi-Riemannian Geometry with Applications to Relativity*. Academic Press, New York, London 1983.
9. Oubiña, J. A. New classes of almost contact metric structures. *Publ. Math. Debrecen*, 1985, **32**, 187–193.
10. Özgür, C. and Sular, S. Warped products with semi-symmetric non-metric connections. *Arab. J. Sci. Eng.*, 2011, **36**, 461–473.
11. Sular, S. and Özgür, C. Warped product manifolds with semi-symmetric metric connections. *Taiwan. J. Math.*, 2011, **15**, 1701–1719.
12. Sular, S. and Özgür, C. Generalized Sasakian space forms with semi-symmetric metric connections. *An. Ştiinţ. Univ. Al. I. Cuza Iaşi. Mat. (N.S.)*, forthcoming.
13. Tricerri, F. and Vanhecke, L. Curvature tensors on almost Hermitian manifolds. *T. Am. Math. Soc.*, 1981, **267**, 365–398.
14. Vanhecke, L. Almost Hermitian manifolds with J-invariant Riemann curvature tensor. *Rendiconti del Seminario Mathematico della Universitá e Politecnico di Torino*, 1975–76, **34**, 487–498.
15. Yano, K. On semi-symmetric metric connections. *Rev. Roumaine Math. Pures Appl.*, 1970, **15**, 1579–1586.

## Poolsümmeetrilise mittemeetrilise seostusega üldistatud Sasaki ruumivormid

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On tutvustatud poolsümmeetrilise mittemeetrilise seostusega üldistatud Sasaki ruumivorme. On defineeritud poolsümmeetrilise mittemeetrilise seostusega üldistatud Sasaki ruumivormi mõiste, tõestatud olemasoluteoreem ja toodud selliste ruumivormide näiteid.