KYUNGPOOK Math. J. 55(2015), 893-907
http://dx.doi.org/10.5666/KMJ.2015.55.4.893
pISSN 1225-6951 eISSN 0454-8124

Nearly k-th Partial Ternary Quadratic $*$-Derivations

Berna Arslan* and Hülya Inceboz
Department of Mathematics, Adnan Menderes University, Aydın, Turkey
e-mail : byorganci@adu.edu.tr *and* hinceboz@adu.edu.tr

Ali Güven
Department of Mathematics, Balikesir University, Balikesir, Turkey
e-mail : guvennali@gmail.com

Abstract. The Hyers-Ulam-Rassias stability of the k-th partial ternary quadratic derivations is investigated in non-Archimedean Banach ternary algebras and non-Archimedean C^*-ternary algebras by using the fixed point theorem.

1. Introduction and Preliminaries

The stability of functional equations was started in 1940 with a problem raised by S. M. Ulam [24], concerning group homomorphisms:

Let $(G_1, *)$ be a group and let $(G_2, \circ, d)$ be a metric group with the metric $d(.,.)$. Given $\epsilon > 0$, does there exist a $\delta(\epsilon) > 0$ such that if a function $f : G_1 \to G_2$ satisfies the inequality

$$d(f(x * y), f(x) \circ f(y)) < \delta$$

for all $x, y \in G_1$, then there exists a homomorphism $h : G_1 \to G_2$ with $d(f(x), h(x)) < \epsilon$ for all $x \in G_1$?

In other words, we are looking for situations when the homomorphisms are stable, i.e., if a mapping is almost a homomorphism, then there exists a true homomorphism near it.

In 1941, Hyers [8] gave a first affirmative answer to the question of Ulam for the case of approximate additive mappings under the assumption that G_1 and G_2

* Corresponding Author.

Received January 22, 2015; revised May 15, 2015; accepted May 27, 2015.
2010 Mathematics Subject Classification: Primary 39B52; Secondary 46L05, 47B48, 39B82, 47H10, 13N15, 46L57, 17A40.
Key words and phrases: Partial ternary quadratic derivation, non-Archimedean ternary algebra, Hyers-Ulam-Rassias stability, fixed point alternative, non-Archimedean C^*-ternary algebra.

are Banach spaces. In 1978, Th. M. Rassias [21] extended the theorem of Hyers by considering the stability problem with unbounded Cauchy difference inequality

$$\|f(x+y)-f(x)-f(y)\| \leq \epsilon(\|x\|^p+\|y\|^p) \qquad (\epsilon \geq 0, p \in [0,1)).$$

Namely, he has proved the following:

Theorem 1.1.([21]) *Let E_1, E_2 be Banach spaces. If $f : E_1 \to E_2$ satisfies the inequality*

$$\|f(x+y)-f(x)-f(y)\| \leq \epsilon(\|x\|^p+\|y\|^p)$$

for all $x, y \in E_1$, where ϵ and p are constants with $\epsilon \geq 0$ and $0 \leq p < 1$, then there exists a unique additive mapping $A : E_1 \to E_2$ such that

$$\|f(x)-A(x)\| \leq \frac{2\epsilon}{2-2^p}\|x\|^p$$

for all $x \in E_1$. If, moreover, the function $t \mapsto f(tx)$ from $\mathbb{R}$ into E_2 is continuous for each fixed $x \in E_1$, then the mapping A is $\mathbb{R}$-linear.

This result provided a remarkable generalization of the Hyers' theorem. So this kind of stability that was introduced by Th. M. Rassias [21] is called the Hyers-Ulam-Rassias stability of functional equations. In 1994, Găvruta [7] obtained a generalization of Rassias' theorem by replacing the bound $\epsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x, y)$.

The Hyers-Ulam-Rassias stability problems of various functional equations and mappings with more general domains and ranges have been investigated by several mathematicians (see [13]-[17]). We also refer the readers to the books [4],[9] and [22].

The stability result concerning derivations between operator algebras was first obtained by Šemrl in [23]. Park and et al. proved the stability of homomorphisms and derivations in Banach algebras, Banach ternary algebras, C^*-algebras, Lie C^*-algebras and C^*-ternary algebras ([3],[18],[19],[20]).

We recall some basic facts concerning Banach ternary algebras and some preliminary results.

Let A be a linear space over a complex field equipped with a mapping, the so-called ternary product, $[\,] : A \times A \times A \to A$ with $(x, y, z) \mapsto [xyz]$ that is linear in variables x, y, z and satisfies the associative identity, i.e. $[[xyz]uv] = [x[yzu]v] = [xy[zuv]]$ for all $x, y, z, u, v \in A$. The pair $(A, [\,])$ is called a ternary algebra. The ternary algebra $(A, [\,])$ is called unital if it has an identity element, i.e. an element $e \in A$ such that $[xee] = [eex] = x$ for every $x \in A$. A $*$-ternary algebra is a ternary algebra together with a mapping $x \mapsto x^*$ from A into A which satisfies

(i) $(x^*)^* = x$,

(ii) $(\lambda x)^* = \bar{\lambda} x^*$,

(iii) $(x+y)^* = x^* + y^*$,

(iv) $[xyz]^* = [z^* y^* x^*]$

for all $x, y, z \in A$ and all $\lambda \in \mathbb{C}$. In the case that A is unital and e is its unit, we assume that $e^* = e$.

A is a normed ternary algebra if A is a ternary algebra and there exists a norm $\|.\|$ on A which satisfies $\|[xyz]\| \leq \|x\|\|y\|\|z\|$ for all $x, y, z \in A$. If A is a unital ternary algebra with unit element e, then $\|e\| = 1$. By a Banach ternary algebra we mean a normed ternary algebra with a complete norm $\|.\|$. If A is a ternary algebra, $x \in A$ is called central if $[xyz] = [zxy] = [yzx]$ for all $y, z \in A$. The set of all central elements of A is called the center of A which is denoted by $Z(A)$.

If A is $*$-normed ternary algebra and $Z(A) = 0$, then we have $\|x^*\| = \|x\|$. A C^*-ternary algebra is a Banach $*$-ternary algebra if $\|[x^* y x]\| = \|x\|^2 \|y\|$ for all x in A and y in $Z(A)$.

In 2010, Eshaghi and et al. [6] introduced the concept of a partial ternary derivation and proved the Hyers-Ulam-Rassias stability of partial ternary derivations in Banach ternary algebras. Recently, Javadian and et al. [10] established the Hyers-Ulam-Rassias stability of the partial ternary quadratic derivations in Banach ternary algebras by using the direct method.

Let $A_1, \ldots, A_n$ be normed ternary algebras over the complex field $\mathbb{C}$ and let B be a Banach ternary algebra over $\mathbb{C}$. As in [10], a mapping $\delta_k : A_1 \times \ldots \times A_n \to B$ is called a *k-th partial ternary quadratic derivation* if

$$\begin{aligned} & \delta_k(x_1, \ldots, a_k + b_k, \ldots, x_n) + \delta_k(x_1, \ldots, a_k - b_k, \ldots, x_n) \\ = \; & 2\delta_k(x_1, \ldots, a_k, \ldots, x_n) + 2\delta_k(x_1, \ldots, b_k, \ldots, x_n) \end{aligned}$$

and there exists a mapping $g_k : A_k \to B$ such that

$$\begin{aligned} & \delta_k(x_1, \ldots, [a_k b_k c_k], \ldots, x_n) = [g_k(a_k) g_k(b_k) \delta_k(x_1, \ldots, c_k, \ldots, x_n)] \\ & + [g_k(a_k) \delta_k(x_1, \ldots, b_k, \ldots, x_n) g_k(c_k)] + [\delta_k(x_1, \ldots, a_k, \ldots, x_n) g_k(b_k) g_k(c_k)] \end{aligned}$$

for all $a_k, b_k, c_k \in A_k$ and all $x_i \in A_i$ $(i \neq k)$.

If, δ_k satisfies the additional condition

$$\delta_k(x_1, \ldots, a_k^*, \ldots, x_n) = (\delta_k(x_1, \ldots, a_k, \ldots, x_n))^*$$

for all $a_k \in A_k$, $x_i \in A_i$ $(i \neq k)$, then δ_k is called a *k-th partial ternary quadratic $*$-derivation.*

Let $\mathbb{K}$ denote a field and $|.|$ be a function (valuation absolute) from $\mathbb{K}$ into $[0, \infty)$. By a *non-Archimedean valuation* we mean a function $|.|$ that satisfies the conditions $|r| = 0$ if and only if $r = 0$, $|rs| = |r||s|$ and the strong triangle inequality, namely,

$$|r + s| \leq \max\{|r|, |s|\} \leq |r| + |s|$$

for all $r, s \in \mathbb{K}$. The associated field $\mathbb{K}$ is referred to as a non-Archimedean field. Clearly, $|1| = |-1| = 1$ and $|n| \leq 1$ for all $n \in \mathbb{N}$. By the trivial valuation we mean the mapping $|.|$ taking everything except 0 into 1 and $|0| = 0$.

Let X be a vector space over a field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|.|$. A function $\|.\| : X \to \mathbb{R}$ is called a *non-Archimedean norm* if it satisfies the following conditions:

(i) $\|x\| = 0$ if and only if $x = 0$;

(ii) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$, $x \in X$;

(iii) $\|x + y\| \leq \max\{\|x\|, \|y\|\}$ for all $x, y \in X$ (strong triangle inequality).

Then, $(X, \|.\|)$ is called a *non-Archimedean normed space.*

From the fact that

$$\|x_n - x_m\| \leq \max\{\|x_{j+1} - x_j\| : m \leq j \leq n - 1\} \qquad (n > m)$$

holds, a sequence $\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence if and only if $\{x_{n+1} - x_n\}_{n\in\mathbb{N}}$ converges to zero in a non-Archimedean normed space. By a complete non-Archimedean normed space, we mean one in which every Cauchy sequence is convergent.

Suppose that p is a prime number. For any nonzero rational number x, there exists a unique integer $n_x \in \mathbb{Z}$ such that $x = (a/b)p^{n_x}$, where a and b are integers not divisible by p. Define the *p-adic absolute value* $|x|_p := p^{-n_x}$. Then $|.|$ is a non-Archimedean norm on $\mathbb{Q}$ with the p-adic absolute value $|.|_p$. The completion of $\mathbb{Q}$ with respect to $|.|$ is denoted by $\mathbb{Q}_p$, which is called the *p-adic number field.*

By a *non-Archimedean Banach ternary algebra* we mean a complete non-Archimedean vector space A equipped with a ternary product $(x, y, z) \mapsto [xyz]$ of A^3 into A which is $\mathbb{K}$-linear in each variables and associative in the sense that

$$[xy[zwv]] = [x[yzw]v] = [[xyz]wv]$$

and satisfies the following

$$\|[xyz]\| \leq \|x\|\|y\|\|z\|$$

for all $x, y, z, w, v \in A$. A *non-Archimedean C^*-ternary algebra* is a non-Archimedean Banach $*$-ternary algebra A if $\|[x^*yx]\| = \|x\|^2\|y\|$ for all $x \in A$ and $y \in Z(A)$.

We now recall a fundamental result in fixed point theory. Let X be a nonempty set. A function $d : X \times X \to [0, \infty]$ is called a *non-Archimedean generalized metric* on X if and only if d satisfies

(i) $d(x,y)=0$ if and only if $x=y$,

(ii) $d(x,y)=d(y,x)$,

(iii) $d(x,z)\le \max\{d(x,y),d(y,z)\}$

for all $x,y,z\in X$. Then (X,d) is called a *non-Archimedean generalized metric space.*

Now, we need the following fixed point theorem (see [5]):

Theorem 1.2.(Non-Archimedean Alternative Contraction Principle) *Let (X,d) be a non-Archimedean generalized complete metric space and $\Lambda: X\to X$ is a strictly contractive mapping, that is,*

$$d(\Lambda x,\Lambda y)\le Ld(x,y) \qquad (x,y\in X)$$

with the Lipschitz constant $L<1$. If there exists a nonnegative integer n_0 such that $d(\Lambda^{n_0+1}x,\Lambda^{n_0}x)<\infty$ for some $x\in X$, then the following statements are true:

(i) *The sequence $\{\Lambda^n x\}$ converges to a fixed point x^* of Λ;*

(ii) *x^* is a unique fixed point of Λ in*

$$X^*=\{y\in X \mid d(\Lambda^{n_0}x,y)<\infty\};$$

(iii) *If $y\in X^*$, then*

$$d(y,x^*)\le d(\Lambda y,y).$$

In this paper, using the fixed point method, we prove the Hyers-Ulam-Rassias stability and superstability of partial ternary quadratic derivations in non-Archimedean Banach ternary algebras and non-Archimedean C^*-ternary algebras.

2. Stability of Partial Ternary Quadratic Derivations in Non-Archimedean Banach Ternary Algebras

Throughout this section, we assume that $A_1,\dots,A_n$ are non-Archimedean ternary normed algebras over a non-Archimedean field $\mathbb{K}$, and B is a non-Archimedean Banach ternary algebra over $\mathbb{K}$. We denote that $0_k, 0_B$ are zero elements of A_k, B, respectively.

Theorem 2.1. *Let $F_k: A_1\times\ldots\times A_n\to B$ be a mapping with $F_k(x_1,\dots,0_k,\dots,x_n)=0_B$. Assume that there exist a function $\varphi_k: A_k^3\to[0,\infty)$ and a quadratic mapping $g_k: A_k\to B$ such that*

$$\begin{aligned}(2.1)\qquad &\|F_k(x_1,\dots,a_k+b_k,\dots,x_n)+F_k(x_1,\dots,a_k-b_k,\dots,x_n)\\ &-2F_k(x_1,\dots,a_k,\dots,x_n)-2F_k(x_1,\dots,b_k,\dots,x_n)\|\le\varphi_k(a_k,b_k,0_k)\end{aligned}$$

and

$$\begin{aligned} &\|F_k(x_1,\ldots,[a_kb_kc_k],\ldots,x_n)-[g_k(a_k)g_k(b_k)F_k(x_1,\ldots,c_k,\ldots,x_n)] \\ &-[g_k(a_k)F_k(x_1,\ldots,b_k,\ldots,x_n)g_k(c_k)]-[F_k(x_1,\ldots,a_k,\ldots,x_n)g_k(b_k)g_k(c_k)]\| \\ &\leq \varphi_k(a_k,b_k,c_k) \end{aligned} \tag{2.2}$$

for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. *Suppose that there exist a natural number* $t \in \mathbb{K}$ *and* $L \in (0,1)$ *such that*

$$\varphi_k(t^{-1}a_k, t^{-1}b_k, t^{-1}c_k) \leq |t|^{-2}L\varphi_k(a_k,b_k,c_k) \tag{2.3}$$

for all $a_k, b_k, c_k \in A_k$. *Then there exists a unique* k*-th partial ternary quadratic derivation* $\delta_k : A_1 \times \cdots \times A_n \to B$ *such that*

$$\|F_k(x_1,\ldots,x_n)-\delta_k(x_1,\ldots,x_n)\| \leq |t|^{-2}L\psi(x_k) \tag{2.4}$$

for all $x_i \in A_i$ $(i = 1,2,\ldots,n)$, *where*

$$\begin{aligned} \psi(x_k) \ := \ & \max\{\varphi_k(0_k,0_k,0_k),\varphi_k(x_k,x_k,0_k),\varphi_k(2x_k,x_k,0_k), \\ & \ldots,\varphi_k((k-1)x_k,x_k,0_k)\}. \end{aligned} \tag{2.5}$$

Proof. By (2.3), one can show that

$$\lim_{m\to\infty} |t|^{2m}\varphi_k(t^{-m}a_k, t^{-m}b_k, t^{-m}c_k) = 0 \tag{2.6}$$

for all $a_k, b_k, c_k \in A_k$. One can use induction on m to show that

$$\begin{aligned} &\|F_k(x_1,\ldots,mx_k,\ldots,x_n)-m^2F_k(x_1,\ldots,x_k,\ldots,x_n)\| \\ &\leq \max\{\varphi_k(0_k,0_k,0_k),\varphi_k(x_k,x_k,0_k),\varphi_k(2x_k,x_k,0_k), \\ &\ldots,\varphi_k((m-1)x_k,x_k,0_k)\} \end{aligned} \tag{2.7}$$

for all $x_i \in A_i$ $(i = 1,2,\ldots,n)$ and all non-negative integers m. Indeed, putting $a_k = b_k = x_k$ in (2.1), we get

$$\begin{aligned} &\|F_k(x_1,\ldots,2x_k,\ldots,x_n)-4F_k(x_1,\ldots,x_k,\ldots,x_n)\| \\ &\leq \ \max\{\varphi_k(0_k,0_k,0_k),\varphi_k(x_k,x_k,0_k)\} \end{aligned} \tag{2.8}$$

for all $x_i \in A_i$, $i = 1,2,\ldots,n$. This proves (2.7) hold for $m = 2$. Let (2.7) holds for $m = 1,2,\ldots,j$. Replacing a_k, b_k with jx_k, x_k, respectively, in (2.1), we obtain

$$\begin{aligned} &\|F_k(x_1,\ldots,(j+1)x_k,\ldots,x_n)+F_k(x_1,\ldots,(j-1)x_k,\ldots,x_n) \\ &-2F_k(x_1,\ldots,jx_k,\ldots,x_n)-2F_k(x_1,\ldots,x_k,\ldots,x_n)\| \\ &\leq \ \max\{\varphi_k(0_k,0_k,0_k),\varphi_k(jx_k,x_k,0_k)\}. \end{aligned} \tag{2.9}$$

Since

$$\begin{aligned}
& F_k(x_1,\dots,(j+1)x_k,\dots,x_n)+F_k(x_1,\dots,(j-1)x_k,\dots,x_n) \\
& \qquad -2F_k(x_1,\dots,jx_k,\dots,x_n)-2F_k(x_1,\dots,x_k,\dots,x_n) \\
& = F_k(x_1,\dots,(j+1)x_k,\dots,x_n)-(j+1)^2F_k(x_1,\dots,x_k,\dots,x_n) \\
& \qquad +F_k(x_1,\dots,(j-1)x_k,\dots,x_n)-(j-1)^2F_k(x_1,\dots,x_k,\dots,x_n) \\
& \qquad -2[F_k(x_1,\dots,jx_k,\dots,x_n)-j^2F_k(x_1,\dots,x_k,\dots,x_n)] \qquad (2.10)
\end{aligned}$$

for all $x_i \in A_i$ $(i=1,2,\dots,n)$, it follows from induction hypothesis and (2.9) that for all $x_i \in A_i$ $(i=1,2,\dots,n)$,

$$\begin{aligned}
& \|F_k(x_1,\dots,(j+1)x_k,\dots,x_n)-(j+1)^2F_k(x_1,\dots,x_k,\dots,x_n)\| \qquad (2.11) \\
& \leq \max\{\|F_k(x_1,\dots,(j+1)x_k,\dots,x_n)+F_k(x_1,\dots,(j-1)x_k,\dots,x_n) \\
& \quad -2F_k(x_1,\dots,jx_k,\dots,x_n)-2F_k(x_1,\dots,x_k,\dots,x_n)\|, \\
& \quad \|F_k(x_1,\dots,(j-1)x_k,\dots,x_n)-(j-1)^2F_k(x_1,\dots,x_k,\dots,x_n)\|, \\
& \quad |2|\|j^2F_k(x_1,\dots,x_k,\dots,x_n)-F_k(x_1,\dots,jx_k,\dots,x_n)\|\} \\
& \leq \max\{\varphi_k(0_k,0_k,0_k),\varphi_k(x_k,x_k,0_k),\varphi_k(2x_k,x_k,0_k),\dots,\varphi_k(jx_k,x_k,0_k)\}.
\end{aligned}$$

This proves (2.7) for all $m \geq 2$. In particular, for all $x_i \in A_i$ $(i=1,2,\dots,n)$

$$\|F_k(x_1,\dots,tx_k,\dots,x_n)-t^2F_k(x_1,\dots,x_k,\dots,x_n)\| \leq \psi(x_k). \qquad (2.12)$$

Replacing x_k by $t^{-1}x_k$ in (2.12), we get

$$\|F_k(x_1,\dots,x_k,\dots,x_n)-t^2F_k(x_1,\dots,t^{-1}x_k,\dots,x_n)\| \leq \psi(t^{-1}x_k) \qquad (2.13)$$

for all $x_i \in A_i$ $(i=1,2,\dots,n)$.

Let us define a set X of all functions $H_k : A_1\times\ldots\times A_n \to B$ by

$$\begin{aligned}
X = \{ & H_k : A_1\times\ldots\times A_n \to B, \quad H_k(x_1,\dots,0_k,\dots,x_n)=0_B, \\
& x_i \in A_i, \quad i=1,2,\dots,n\}
\end{aligned}$$

and introduce ρ on X as follows:

$$\begin{aligned}
\rho(F_k,H_k) := \inf\{ & C\in(0,\infty) : \|F_k(x_1,\dots,x_k,\dots,x_n) \qquad (2.14) \\
& -H_k(x_1,\dots,x_k,\dots,x_n)\| \leq C\psi(x_k), \quad \forall x_i \in A_i, \quad i=1,2,\dots,n)\}.
\end{aligned}$$

It is easy to see that ρ defines a generalized non-Archimedean complete metric on X (see [1],[2] and [12]). Now we consider the function $J : X \to X$ defined by

$$J(H_k)(x_1,\dots,x_k,\dots,x_n) := t^2 H_k(x_1,\dots,t^{-1}x_k,\dots,x_n)$$

for all $x_i \in A_i$ $(i = 1,2,\dots,n)$ and $H_k \in X$. Then J is strictly contractive on X, in fact if for all $x_i \in A_i$ $(i = 1,2,\dots,n)$,

$$\|F_k(x_1,\dots,x_k,\dots,x_n) - H_k(x_1,\dots,x_k,\dots,x_n)\| \le C\psi(x_k) \tag{2.15}$$

then by (2.3),

$$\begin{aligned} &\|J(F_k)(x_1,\dots,x_k,\dots,x_n) - J(H_k)(x_1,\dots,x_k,\dots,x_n)\| \\ = \ &|t|^2\|F_k(x_1,\dots,t^{-1}x_k,\dots,x_n) - H_k(x_1,\dots,t^{-1}x_k,\dots,x_n)\| \\ \le \ &C|t|^2\psi(t^{-1}x_k) \le CL\psi(x_k) \qquad\qquad (x_k \in A_k). \end{aligned} \tag{2.16}$$

So it follows that

$$\rho(J(F_k), J(H_k)) \le L\rho(F_k, H_k) \qquad (F_k, H_k \in X). \tag{2.17}$$

Hence, J is a strictly contractive mapping with Lipschitz constant L. Also we obtain by (2.13) that

$$\begin{aligned} &\|J(F_k)(x_1,\dots,x_k,\dots,x_n) - F_k(x_1,\dots,x_k,\dots,x_n)\| \\ = \ &\|t^2F_k(x_1,\dots,t^{-1}x_k,\dots,x_n) - F_k(x_1,\dots,x_k,\dots,x_n)\| \\ \le \ &\psi(t^{-1}x_k) \le |t|^{-2}L\psi(x_k) \end{aligned} \tag{2.18}$$

for all $x_i \in A_i$ $(i = 1,2,\dots,n)$. This means that $\rho(J(F_k), F_k) \le |t|^{-2}L < \infty$. Now, from Theorem 1.2, it follows that J has a unique fixed point $\delta_k : A_1 \times \dots \times A_n \to B$ in the set

$$U_k = \{H_k \in X : \rho(H_k, J(F_k)) < \infty\}$$

and for each $x_i \in A_i$ $(i = 1,2,\dots,n)$,

$$\begin{aligned} \delta_k(x_1,\dots,x_n) &:= \lim_{m\to\infty} J^m(F_k(x_1,\dots,x_k,\dots,x_n)) \\ &= \lim_{m\to\infty} t^{2m}(F_k(x_1,\dots,t^{-m}x_k,\dots,x_n)). \end{aligned} \tag{2.19}$$

Then we obtain from (2.1) and (2.6) that

$$\begin{aligned} &\|\delta_k(x_1,\dots,a_k+b_k,\dots,x_n) + \delta_k(x_1,\dots,a_k-b_k,\dots,x_n) \\ &\quad -2\delta_k(x_1,\dots,a_k,\dots,x_n) - 2\delta_k(x_1,\dots,b_k,\dots,x_n)\| \\ = \ &\lim_{m\to\infty} |t|^{2m}\|F_k(x_1,\dots,t^{-m}(a_k+b_k),\dots,x_n) + F_k(x_1,\dots,t^{-m}(a_k-b_k),\dots,x_n) \\ &\quad -2F_k(x_1,\dots,t^{-m}a_k,\dots,x_n) - 2F_k(x_1,\dots,t^{-m}b_k,\dots,x_n)\| \\ \le \ &\lim_{m\to\infty} |t|^{2m}\max\{\varphi_k(0_k,0_k,0_k), \varphi_k(t^{-m}a_k, t^{-m}b_k, 0_k)\} = 0 \end{aligned}$$

for each $a_k, b_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. This shows that δ_k is partial quadratic. It follows from Theorem 1.2 that

$$\rho(F_k, \delta_k) \leq \rho(J(F_k), F_k),$$

that is, δ_k is a partial quadratic mapping which satisfies (2.4).

Now, replacing a_k, b_k, c_k with $t^{-m}a_k, t^{-m}b_k, t^{-m}c_k$, respectively, in (2.2), we obtain

$$\begin{aligned}
&\|F_k(x_1, \ldots, [(t^{-3m})a_k b_k c_k], \ldots, x_n) \\
&\quad -[t^{-2m}g_k(a_k)t^{-2m}g_k(b_k)F_k(x_1, \ldots, t^{-m}c_k, \ldots, x_n)] \\
&\quad -[t^{-2m}g_k(a_k)F_k(x_1, \ldots, t^{-m}b_k, \ldots, x_n)t^{-2m}g_k(c_k)] \\
&\quad -[F_k(x_1, \ldots, t^{-m}a_k, \ldots, x_n)t^{-2m}g_k(b_k)t^{-2m}g_k(c_k)]\| \\
\leq \;& \varphi_k(t^{-m}a_k, t^{-m}b_k, t^{-m}c_k).
\end{aligned}$$

Then we have

$$\begin{aligned}
&\|t^{6m}F_k(x_1, \ldots, t^{-3m}[a_k b_k c_k], \ldots, x_n) \\
&\quad -t^{6m}[t^{-2m}g_k(a_k)t^{-2m}g_k(b_k)F_k(x_1, \ldots, t^{-m}c_k, \ldots, x_n)] \\
&\quad -t^{6m}[t^{-2m}g_k(a_k)F_k(x_1, \ldots, t^{-m}b_k, \ldots, x_n)t^{-2m}g_k(c_k)] \\
&\quad -t^{6m}[F_k(x_1, \ldots, t^{-m}a_k, \ldots, x_n)t^{-2m}g_k(b_k)t^{-2m}g_k(c_k)]\| \\
\leq \;& |t|^{6m}\varphi_k(t^{-m}a_k, t^{-m}b_k, t^{-m}c_k)
\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. Taking the limit as $m \to \infty$ in above inequality, we obtain from (2.6) that

$$\begin{aligned}
&\|\lim_{m\to\infty} t^{6m}F_k(x_1, \ldots, t^{-3m}[a_k b_k c_k], \ldots, x_n) \\
&\quad -[g_k(a_k)g_k(b_k)\lim_{m\to\infty} t^{2m}F_k(x_1, \ldots, t^{-m}c_k, \ldots, x_n)] \\
&\quad -[g_k(a_k)\lim_{m\to\infty} t^{2m}F_k(x_1, \ldots, t^{-m}b_k, \ldots, x_n)g_k(c_k)] \\
&\quad -[\lim_{m\to\infty} t^{2m}F_k(x_1, \ldots, t^{-m}a_k, \ldots, x_n)g_k(b_k)g_k(c_k)]\| \\
\leq \;& \lim_{m\to\infty} |t|^{6m}\varphi_k(t^{-m}a_k, t^{-m}b_k, t^{-m}c_k) = 0
\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. Since g_k is a quadratic mapping, we have

$$\begin{aligned}
&\delta_k(x_1, \ldots, [a_k b_k c_k], \ldots, x_n) = [g_k(a_k)g_k(b_k)\delta_k(x_1, \ldots, c_k, \ldots, x_n)] \\
&+[g_k(a_k)\delta_k(x_1, \ldots, b_k, \ldots, x_n)g_k(c_k)] + [\delta_k(x_1, \ldots, a_k, \ldots, x_n)g_k(b_k)g_k(c_k)]
\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$ and all $x_i \in A_i$ $(i \neq k)$. Thus $\delta_k : A_1 \times \cdots \times A_n \to B$ is a k-th partial ternary quadratic derivation, satisfying (2.4), as desired. □

In the following corollaries, $\mathbb{Q}_p$ is the p-adic number field, where $p > 2$ is a prime number.

By Theorem 2.1, we show the following Hyers-Ulam-Rassias stability of partial ternary quadratic derivations on non-Archimedean Banach ternary algebras.

Corollary 2.2. *Let $A_1, \ldots, A_n$ be non-Archimedean ternary normed algebras over $\mathbb{Q}_p$ with norm $\|.\|$ and $(B, \|.\|_B)$ be a non-Archimedean Banach ternary algebra over $\mathbb{Q}_p$. Suppose that $F_k : A_1 \times \cdots \times A_n \to B$ is a mapping and $g_k : A_k \to B$ is a quadratic mapping such that for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$,*

$$\begin{aligned} (2.20)\quad & \|F_k(x_1, \ldots, a_k + b_k, \ldots, x_n) + F_k(x_1, \ldots, a_k - b_k, \ldots, x_n) \\ & -2F_k(x_1, \ldots, a_k, \ldots, x_n) - 2F_k(x_1, \ldots, b_k, \ldots, x_n)\|_B \leq \theta(\|a_k\|^r + \|b_k\|^r) \end{aligned}$$

and

$$\begin{aligned} (2.21)\quad & \|F_k(x_1, \ldots, [a_k b_k c_k], \ldots, x_n) - [g_k(a_k) g_k(b_k) F_k(x_1, \ldots, c_k, \ldots, x_n)] \\ & -[g_k(a_k) F_k(x_1, \ldots, b_k, \ldots, x_n) g_k(c_k)] - [F_k(x_1, \ldots, a_k, \ldots, x_n) g_k(b_k) g_k(c_k)]\|_B \\ \leq \quad & \theta(\|a_k\|^r + \|b_k\|^r + \|c_k\|^r) \end{aligned}$$

for some $\theta > 0$ and $r \geq 0$ with $r < 2$. Then there exists a unique k-th partial ternary quadratic derivation $\delta_k : A_1 \times \cdots \times A_n \to B$ such that

$$\|F_k(x_1, \ldots, x_n) - \delta_k(x_1, \ldots, x_n)\|_B \leq 2\theta p^r \|x_k\|^r$$

holds for all $x_i \in A_i$ $(i = 1, 2, \ldots, n)$.

Proof. By (2.20), we have $F_k(x_1, \ldots, 0_k, \ldots, x_n) = 0_B$. Let

$$(2.22)\qquad \varphi_k(a_k, b_k, c_k) := \theta(\|a_k\|^r + \|b_k\|^r + \|c_k\|^r),$$

for all $a_k, b_k, c_k \in A_k$. Then by replacing a_k, b_k, c_k with $p^{-1}a_k$, $p^{-1}b_k$, $p^{-1}c_k$, respectively, in (2.22), we have

$$\begin{aligned} \varphi_k(p^{-1}a_k, p^{-1}b_k, p^{-1}c_k) &= \theta(\|p^{-1}a_k\|^r + \|p^{-1}b_k\|^r + \|p^{-1}c_k\|^r) \\ &= \theta(|p^{-1}|^r \|a_k\|^r + |p^{-1}|^r \|b_k\|^r + |p^{-1}|^r \|c_k\|^r) \\ &= \theta p^r (\|a_k\|^r + \|b_k\|^r + \|c_k\|^r) \\ &= p^r \varphi_k(a_k, b_k, c_k) \end{aligned}$$

for all $a_k, b_k, c_k \in A_k$, since $|p^{-1}| = p$ by the definition of the p-adic absolute value. Also,

$$\begin{aligned} \psi(x_k) \quad := \quad & \max\{\varphi_k(0_k, 0_k, 0_k), \varphi_k(x_k, x_k, 0_k), \varphi_k(2x_k, x_k, 0_k), \\ & \ldots, \varphi_k((p-1)x_k, x_k, 0_k)\} = 2\theta \|x_k\|^r \end{aligned}$$

for all $x_k \in A_k$.

In Theorem 2.1, by putting $L := p^{r-2} < 1$, we obtain the conclusion of the theorem. □

Similarly, we can obtain the following theorem. So, we will omit the proof.

Theorem 2.3. *Let* $F_k : A_1 \times \ldots \times A_n \to B$ *be a mapping with* $F_k(x_1, \ldots, 0_k, \ldots, x_n) = 0_B$. *Assume that there exist a function* $\varphi_k : A_k^3 \to [0, \infty)$ *and a quadratic mapping* $g_k : A_k \to B$ *such that*

$$\begin{aligned}(2.23)\ &\|F_k(x_1, \ldots, a_k + b_k, \ldots, x_n) + F_k(x_1, \ldots, a_k - b_k, \ldots, x_n)\\ &-2F_k(x_1, \ldots, a_k, \ldots, x_n) - 2F_k(x_1, \ldots, b_k, \ldots, x_n)\| \leq \varphi_k(a_k, b_k, 0_k)\end{aligned}$$

and

$$\begin{aligned}(2.24)\ &\|F_k(x_1, \ldots, [a_k b_k c_k], \ldots, x_n) - [g_k(a_k) g_k(b_k) F_k(x_1, \ldots, c_k, \ldots, x_n)]\\ &-[g_k(a_k) F_k(x_1, \ldots, b_k, \ldots, x_n) g_k(c_k)] - [F_k(x_1, \ldots, a_k, \ldots, x_n) g_k(b_k) g_k(c_k)]\|\\ &\leq \varphi_k(a_k, b_k, c_k)\end{aligned}$$

for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. *If there exist a natural number* $t \in \mathbb{K}$ *and* $0 < L < 1$ *such that*

$$\varphi_k(ta_k, tb_k, tc_k) \leq |t|^2 L \varphi_k(a_k, b_k, c_k) \tag{2.25}$$

for all $a_k, b_k, c_k \in A_k$, *then there exists a unique k-th partial ternary quadratic derivation* $\delta_k : A_1 \times \cdots \times A_n \to B$ *such that*

$$\|F_k(x_1, \ldots, x_n) - \delta_k(x_1, \ldots, x_n)\| \leq |t|^2 L \psi(x_k) \tag{2.26}$$

for all $x_i \in A_i$ $(i = 1, 2, \ldots, n)$, *where*

$$\begin{aligned}(2.27)\quad \psi(x_k) \ := \ &\max\{\varphi_k(0_k, 0_k, 0_k), \varphi_k(x_k, x_k, 0_k), \varphi_k(2x_k, x_k, 0_k),\\ &\ldots, \varphi_k((k-1)x_k, x_k, 0_k)\}.\end{aligned}$$

The following corollary is similar to Corollary 2.2 for the case where $r > 2$.

Corollary 2.4. *Let* $A_1, \ldots, A_n$ *be non-Archimedean ternary normed algebras over* $\mathbb{Q}_p$ *with norm* $\|.\|$ *and* $(B, \|.\|_B)$ *be a non-Archimedean Banach ternary algebra over* $\mathbb{Q}_p$. *Suppose that* $F_k : A_1 \times \cdots \times A_n \to B$ *is a mapping and* $g_k : A_k \to B$ *is a quadratic mapping such that for all* $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$,

$$\begin{aligned}(2.28)\ &\|F_k(x_1, \ldots, a_k + b_k, \ldots, x_n) + F_k(x_1, \ldots, a_k - b_k, \ldots, x_n)\\ &-2F_k(x_1, \ldots, a_k, \ldots, x_n) - 2F_k(x_1, \ldots, b_k, \ldots, x_n)\|_B \leq \theta(\|a_k\|^r + \|b_k\|^r)\end{aligned}$$

and

$$\begin{aligned}(2.29)\ &\|F_k(x_1, \ldots, [a_k b_k c_k], \ldots, x_n) - [g_k(a_k) g_k(b_k) F_k(x_1, \ldots, c_k, \ldots, x_n)]\\ &-[g_k(a_k) F_k(x_1, \ldots, b_k, \ldots, x_n) g_k(c_k)] - [F_k(x_1, \ldots, a_k, \ldots, x_n) g_k(b_k) g_k(c_k)]\|_B\\ &\leq \theta(\|a_k\|^r + \|b_k\|^r + \|c_k\|^r)\end{aligned}$$

for some $\theta > 0$ and $r \geq 0$ with $r > 2$. Then there exists a unique k-th partial ternary quadratic derivation $\delta_k : A_1 \times \cdots \times A_n \to B$ such that

$$\|F_k(x_1, \ldots, x_n) - \delta_k(x_1, \ldots, x_n)\|_B \leq 2\theta p^{-r} \|x_k\|^r$$

holds for all $x_i \in A_i$ $(i = 1, 2, \ldots, n)$.

Proof. From (2.28), we have $F_k(x_1, \ldots, 0_k, \ldots, x_n) = 0_B$. By putting $\varphi_k(a_k, b_k, c_k) := \theta(\|a_k\|^r + \|b_k\|^r + \|c_k\|^r)$ and $L := p^{2-r} < 1$ in Theorem 2.3, we get the desired result. □

Moreover, we have the following result for the superstability of k-th partial ternary quadratic derivations.

Corollary 2.5. *Let r, s, t and θ be real numbers such that $r + s + t < -2$ and $\theta \in (0, \infty)$. Let $A_1, \ldots, A_n$ be non-Archimedean ternary normed algebras over $\mathbb{Q}_p$ with norm $\|.\|$ and $(B, \|.\|_B)$ be a non-Archimedean Banach ternary algebra over $\mathbb{Q}_p$. Assume that $F_k : A_1 \times \cdots \times A_n \to B$ is a mapping and $g_k : A_k \to B$ is a quadratic mapping such that*

$$\|F_k(x_1, \ldots, a_k + b_k, \ldots, x_n) + F_k(x_1, \ldots, a_k - b_k, \ldots, x_n)$$
$$-2F_k(x_1, \ldots, a_k, \ldots, x_n) - 2F_k(x_1, \ldots, b_k, \ldots, x_n)\|_B \leq \theta(\|a_k\|^r + \|b_k\|^r),$$

and

$$\|F_k(x_1, \ldots, [a_k b_k c_k], \ldots, x_n) - [g_k(a_k) g_k(b_k) F_k(x_1, \ldots, c_k, \ldots, x_n)]$$
$$-[g_k(a_k) F_k(x_1, \ldots, b_k, \ldots, x_n) g_k(c_k)] - [F_k(x_1, \ldots, a_k, \ldots, x_n) g_k(b_k) g_k(c_k)]\|_B$$
$$\leq \theta(\|a_k\|^r \|b_k\|^s \|c_k\|^t)$$

for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. Then F_k is a k-th partial ternary quadratic derivation.

Proof. It follows from Theorem 2.1, by putting

$$\varphi_k(a_k, b_k, c_k) := \theta(\|a_k\|^r \|b_k\|^s \|c_k\|^t)$$

for all $a_k, b_k, c_k \in A_k$. □

We can prove a same result with condition $r + s + t > -2$ by using of Theorem 2.3.

3. Stability of Partial Ternary Quadratic $*$-Derivations in Non-Archimedean C^*-Ternary Algebras

In this section, assume that $A_1, \ldots, A_n$ are non-Archimedean $*$-normed ternary algebras over $\mathbb{C}$, and B is a non-Archimedean C^*-ternary algebra.

Theorem 3.1. *Let $F_k : A_1 \times \cdots \times A_n \to B$ be a mapping with $F_k(x_1, \ldots, 0_k, \ldots, x_n) = 0_B$. Suppose that there exist a function $\varphi_k : A_k^3 \to [0, \infty)$ and a quadratic mapping $g_k : A_k \to B$ such that* (2.1) *and* (2.2) *hold and*

$$\|F_k(x_1, \ldots, a_k^*, \ldots, x_n) - (F_k(x_1, \ldots, a_k, \ldots, x_n))^*\| \leq \varphi_k(a_k, 0_k, 0_k) \tag{3.1}$$

for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$. If there exist a natural number $t \in \mathbb{K}$ and $0 < L < 1$ and (2.3) *holds, then there exists a unique k-th partial ternary quadratic $*$-derivation $\delta_k : A_1 \times \cdots \times A_n \to B$ such that* (2.4) *holds.*

Proof. By the same reasoning as in the proof of Theorem 2.1, there exists a unique k-th partial ternary quadratic derivation $\delta_k : A_1 \times \cdots \times A_n \to B$ satisfying (2.4), given by

$$\delta_k(x_1, \ldots, x_n) := \lim_{m\to\infty} t^{2m}(F_k(x_1, \ldots, t^{-m}x_k, \ldots, x_n)) \tag{3.2}$$

for all $x_i \in A_i$ $(i = 1, 2, \ldots, n)$. Now, we have to show that δ_k is $*$-preserving. So it follows from (3.2) that

$$\begin{aligned}
&\|\delta_k(x_1, \ldots, a_k^*, \ldots, x_n) - (\delta_k(x_1, \ldots, a_k, \ldots, x_n))^*\| \\
&= \lim_{m\to\infty} |t|^{2m}\|F_k(x_1, \ldots, t^{-m}a_k^*, \ldots, x_n) - (F_k(x_1, \ldots, t^{-m}a_k, \ldots, x_n))^*\| \\
&= \lim_{m\to\infty} |t|^{2m}\|F_k(x_1, \ldots, (t^{-m}a_k)^*, \ldots, x_n) - (F_k(x_1, \ldots, t^{-m}a_k, \ldots, x_n))^*\| \\
&\leq \lim_{m\to\infty} |t|^{2m} \max\{\varphi_k(0_k, 0_k, 0_k), \varphi_k(t^{-m}a_k, 0_k, 0_k)\} = 0
\end{aligned}$$

for each $a_k \in A_k$, $x_i \in A_i$ $(i \neq k)$.

Thus $\delta_k : A_1 \times \cdots \times A_n \to B$ is a k-th partial ternary quadratic $*$-derivation satisfying (2.4), as desired. □

Now, we prove the following Hyers-Ulam-Rassias stability problem for k-th partial ternary quadratic $*$-derivations on non-Archimedean C^*-ternary algebras.

Corollary 3.2. *Let $A_1, \ldots, A_n$ be non-Archimedean $*$-normed ternary algebras over $\mathbb{Q}_p$ with norm $\|.\|$ and $(B, \|.\|_B)$ be a non-Archimedean C^*-ternary algebra over $\mathbb{Q}_p$. Suppose that $F_k : A_1 \times \cdots \times A_n \to B$ is a mapping and $g_k : A_k \to B$ is a quadratic mapping such that for all $a_k, b_k, c_k \in A_k$, $x_i \in A_i$ $(i \neq k)$,*

$$\begin{aligned}
&\|F_k(x_1, \ldots, a_k + b_k, \ldots, x_n) + F_k(x_1, \ldots, a_k - b_k, \ldots, x_n) \\
&-2F_k(x_1, \ldots, a_k, \ldots, x_n) - 2F_k(x_1, \ldots, b_k, \ldots, x_n)\|_B \leq \theta(\|a_k\|^r + \|b_k\|^r),
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
&\|F_k(x_1, \ldots, [a_k b_k c_k], \ldots, x_n) - [g_k(a_k)g_k(b_k)F_k(x_1, \ldots, c_k, \ldots, x_n)] \\
&-[g_k(a_k)F_k(x_1, \ldots, b_k, \ldots, x_n)g_k(c_k)] - [F_k(x_1, \ldots, a_k, \ldots, x_n)g_k(b_k)g_k(c_k)]\|_B \\
&\leq \theta(\|a_k\|^r + \|b_k\|^r + \|c_k\|^r)
\end{aligned} \tag{3.4}$$

and

$$\|F_k(x_1, \ldots, a_k^*, \ldots, x_n) - (F_k(x_1, \ldots, a_k, \ldots, x_n))^*\|_B \leq \theta\|a_k\|^r \quad (3.5)$$

for some $\theta > 0$ and $r \geq 0$ with $r < 2$. Then there exists a unique k-th partial ternary quadratic $*$-derivation $\delta_k : A_1 \times \cdots \times A_n \to B$ such that

$$\|F_k(x_1, \ldots, x_n) - \delta_k(x_1, \ldots, x_n)\|_B \leq 2\theta p^r \|x_k\|^r$$

holds for all $x_i \in A_i$ $(i = 1, 2, \ldots, n)$.

Proof. The proof follows from Theorem 3.1, by taking $\varphi_k(a_k, b_k, c_k) := \theta(\|a_k\|^r + \|b_k\|^r + \|c_k\|^r)$ for all $a_k, b_k, c_k \in A_k$ and $L = p^{r-2}$, we get the desired result. □

Moreover, we can prove a same result with condition $r > 2$.

Acknowledgements. The authors would like to thank the referee for the careful and detailed reading of the manuscript and valuable suggestions and comments.

References

[1] L. Cădariu and V. Radu, *Fixed points and the stability of Jensen's functional equation*, J. Inequal Pure Appl. Math., **4(1)**(2003), 1-7.

[2] L. Cădariu and V. Radu, *On the stability of the Cauchy functional equation: a fixed point approach*, Grazer Math. Ber., **346**(2004), 43-52.

[3] Y. Cho, R. Saadati and J. Vahidi, *Approximation of homomorphisms and derivations on non-Archimedean Lie C^*-algebras via fixed point method*, Discrete Dynamics in Nature and Society 2012, **Article ID373904**(2012), 1-9.

[4] P. Czerwik, *Functional Equations and Inequalities in Several Variables*, World Scientific Publishing Company, New Jersey, Hong Kong, Singapore and London, 2002.

[5] J. B. Diaz and B. Margolis, *A fixed point theorem of the alternative for contractions on the generalized complete metric space*, Bull. Amer. Math. Soc., **126**(1968), 305-309.

[6] M. Eshaghi, M. B. Savadkouhi, M. Bidkham, C. Park and J. R. Lee, *Nearly partial derivations on Banach ternary algebras*, J. Math. Stat., **6(4)**(2010), 454-461.

[7] P. Găvruta, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. and Appl., **184**(1994), 431-436.

[8] D. H. Hyers, *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. U. S. A., **27**(1941), 222-224.

[9] D. H. Hyers, G. Isac and Th. M. Rassias, *Stability of Functional Equations in Several Variables*, Birkhäuser, Basel, 1998.

[10] A. Javadian, M. E. Gordji and M. B. Savadkouhi, *Approximately partial ternary quadratic derivations on Banach ternary algebras*, J. Nonlinear Sci. Appl., **4(1)**(2011), 60-69.

[11] T. Miura, G. Hirasawa and S.-E. Takahasi, *A perturbation of ring derivations on Banach algebras*, J. Math. Anal. Appl., **319**(2006), 522-530.

[12] A. K. Mirmostafaee, *Hyers-Ulam stability of cubic mappings in non-Archimedean normed spaces*, Kyungpook Math. J., **50**(2010), 315-327.

[13] M. Mirzavaziri and M. S. Moslehian, *A fixed point approach to stability of a quadratic equation*, Bull. Braz. Math. Soc., **37**(2006), 361-376.

[14] M. S. Moslehian, *Hyers-Ulam-Rassias stability of generalized derivations*, Int. J. Math. Sci., **Article ID 93942**(2006), 1-8.

[15] A. Najati and M. B. Moghimi, *Stability of a functional equation deriving from quadratic and additive functions in quasi-Banach spaces*, J. Math. Anal. Appl., **337**(2008), 399-415.

[16] A. Najati and C. Park, *Hyers-Ulam-Rassias stability of homomorphisms in quasi-Banach algebras associated to the Pexiderized Cauchy functional equation*, J. Math. Anal. Appl., **335**(2007), 763-778.

[17] C. Park, *On the stability of the linear mapping in Banach modules*, J. Math. Anal. Appl., **275**(2002), 711-720.

[18] C. Park, *On an approximate automorphism on a C^*-algebra*, Proc. Amer. Math. Soc., **132**(2004), 1739-1745.

[19] C. Park and J. M. Rassias, *Stability of the Jensen type functional equation in C^*-algebras: a fixed point approach*, Abstr. Appl. Anal., 2009, **Article ID 360432**(2009), 1-17.

[20] C. Park and T. M. Rassias, *Homomorphisms in C^*-ternary algebras and JB^*-triples*, J. Math. Anal. Appl., **337**(2008), 13-20.

[21] Th. M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72**(1978), 297-300.

[22] Th. M. Rassias, *Functional Equations, Inequalities and Applications*, Kluwer Academic Publishers Co., Dordrecht, Boston, London, 2003.

[23] P. Šemrl, *The functional equation of multiplicative derivation is superstable on standard operator algebras*, Integ. Equ. Oper. Theory, **18**(1994), 118-122.

[24] S. M. Ulam, *Problems in Modern Mathematics*, Chapter VI, Science ed., Wiley, New York, 1940.