Phenomenological Issues in Supersymmetry with Non-holomorphic Soft Breaking

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Abstract

We present a through discussion of motivations for and phenomenological issues in supersymmetric models with minimal matter content and non-holomorphic soft-breaking terms. Using the unification of the gauge couplings and assuming SUSY is broken with non-standard soft terms, we provide semi-analytic solutions of the RGEs for low and high choices of $tan\beta$ which can be used to study the phenomenology in detail.

We also present a generic form of RGIs in mSUGRA framework which can be used to derive new relations in addition to those existing in the literature. Our results are mostly presented with respect to the conventional minimal supersymmetric model for ease of comparison.

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1 Introduction

Supersymmetry is an elegant symmetry for stabilizing the electroweak scale against strong ultraviolet sensitivity of the Higgs sector induced by quantum fluctuations. This symmetry, given that no experiment has yet observed any of the superpartners, cannot be operative at energies below the Fermi scale. This very constraint is saturated by breaking global supersymmetry explicitly via mass parameters $\mathcal{O}(\text{TeV})$ in such a way that the quadratic divergence of the Higgs sector is not regenerated. In more explicit terms, the action density of the minimal supersymmetric model (MSSM) which is based on the superpotential

$$\widehat{W} = h_t\, \widehat{t}_R \widehat{Q}_L \widehat{H}_u + h_b\, \widehat{b}_R \widehat{Q}_L \widehat{H}_d + h_\tau\, \widehat{\tau}_R \widehat{L}_L \widehat{H}_d + \mu \widehat{H}_u \widehat{H}_d \tag{1}$$

as obtained after discarding all Yukawa couplings except those of the heaviest fermions, is augmented by additional terms (see, for instance, [1] for a review)

$$\begin{aligned} & m^2_{H_u} H_u^\dagger H_u + m^2_{H_d} H_d^\dagger H_d + m^2_{t_L} \widetilde{Q}_L^\dagger \widetilde{Q}_L + m^2_{t_R} \widetilde{t}_R^\dagger \widetilde{t}_R + m^2_{b_R} \widetilde{b}_R^\dagger \widetilde{b}_R + m^2_{\tau_L} \widetilde{L}_L^\dagger \widetilde{L}_L + m^2_{\tau_R} \widetilde{\tau}_R^\dagger \widetilde{\tau}_R + \\ & \left[h_t A_t \widetilde{t}_R \widetilde{Q}_L H_u + h_b A_b \widetilde{b}_R \widetilde{Q}_L H_d + h_\tau A_\tau \widetilde{\tau}_R \widetilde{L}_L H_d + \mu' B H_u H_d + \sum_a \frac{M_a}{2} \lambda_a \lambda_a + \text{h.c.} \right] \end{aligned} \tag{2}$$

which contain massive scalars, gauginos as well as a set of triscalar couplings among sfermions and Higgs bosons. The operators in (2) break supersymmetry in such a way that Higgs scalar sector does not develop any quadratic sensitivity to the UV scale.

The soft-breaking terms in (2) do not necessarily represent the most general set of operators. Indeed, one may consider, for instance, triscalar couplings with 'wrong' Higgs as well as bare Higgsino mass terms. Indeed, such terms have recently been shown to occur among flux-induced soft terms within intersecting brane models [2]. Historically, such terms have been classified as hard since they have the potential of regenerating the quadratic divergences [3]. However, this danger occurs only in theories with pure singlets, and in theories like the MSSM they are perfectly soft. Hence, the most general soft-breaking sector must include the operators

$$\mu' \widetilde{H}_u \widetilde{H}_d + h_t A_{t'} \widetilde{t}_R \widetilde{Q}_L H_d^\dagger + h_b A_{b'} \widetilde{b}_R \widetilde{Q}_L H_u^\dagger + h_\tau A'_\tau \widetilde{\tau}_R \widetilde{L}_L H_u^\dagger + \text{h.c.} \tag{3}$$

in addition to those in (2). Clearly, none of these operators mimics those contained in the superpotential (1): they are non-holomorphic soft-breaking operators. Note the structure of the triscalar couplings here; the triscalar couplings in (2) are modified by including the opposite-hypercharge Higgs doublet.

In principle, the theory can contain both μ and μ' couplings. However, in what follows we will follow the viewpoint that the μ parameter is completely soft, that is, μ in the superpotential vanishes. This indeed can happen if the theory is invariant under global chiral symmetries [4] at high scale [5]. What is crucial about vanishing μ is that it automatically solves the μ problem; the theory does not contain a supersymmetric mass parameter with a completely unknown scale. Indeed, in the MSSM stabilization of the μ parameter to the electroweak scale requires the introduction of gauge [6]- or non-gauge [7] extensions in which the vacuum expectation value (VEV) of an MSSM gauge-singlet scalar generates an effective

μ parameter. For these reasons, having a nonvanishing μ' in the soft-breaking sector both solves the μ problem and serves as if there is a μ parameter in the superpotential.

The present work is organized as follows. In Appendix A we give the full list of renormalization group equations (RGEs) for all rigid and soft parameters of the theory (as we hereafter call 'non-holomorphic MSSM' or NHSSM for short). In Appendix B we list down solutions of the RGEs of all model parameters as a function of their boundary values taken at the scale of gauge coupling unification $M_{GUT} \approx 10^{16}$ GeV. An important parameter of the theory is the ratio of the Higgs vacuum expectation values: $\tan\beta \equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle$. In solving the RGEs we will consider low ($\tan\beta = 5$) and high ($\tan\beta = 50$) values of $\tan\beta$ separately. In Sec. 2 we analyze the Z boson mass, in particular, its sensitivity to GUT-scale parameters. Here we will clarify the differences and similarities between the MSSM and NHSSM. In Sec. 3 we will discuss sfermion masses in the MSSM and NHSSM for the purpose of identifying their sensitivities to GUT-scale parameters, in particular, μ_0 and μ'_0. Neutralinos and charginos are considered in the same section. In Sec. 4 we will discuss renormalization group invariants in the MSSM and NHSSM in a comparative manner so as to know what remains scale invariant in two distinct structures. In Sec. 5 we conclude the model.

2 Fine-tuning of the Z boson mass: MSSM vs. NHSSM

It is well known that supersymmetry (SUSY) is not an exact symmerty of Nature, and there is no unique mechanism (gravity mediation, gauge mediation, anomaly mediation, etc.) for realizing its breakdown. From the viewpoint of Non-Standard Soft Breaking in the Minimal Supersymmetric Standard Model (NHSSM), on one hand, its predictions should reproduce the SM agreement with data, ensure unification of gauge couplings at the Grand Unified Theory (GUT) scale with minimal particle content, and on the other, it should preserve naturalness with soft terms [8].

It is expected that in the near future thanks to LHC and its successors, experiments related with superparticle masses and mixings will yield enough information to distinguish between various GUT-models and supersymmetry breaking mechanisms (see *e.g.* [9]). Taking gravity-mediation as the mechanism responsible for SUSY breaking, it is important to explore how the soft terms are induced: holomorphic soft terms of the minimal model or those of the NHSSM with or without R parity violation [10]. In this work we will concentrate on NHSSM with exact R parity deferring the effects of R parity violation to a future work.

Presently, apart from a number of observables in the flavor-chaning neutral current sector, the Z boson mass is the main parameter that relates precision measurements to soft masses. In other words, the soft terms must self-organize so as to reproduce the measured value of the Z boson mass [8]. Hence, it is profitable to analyze M_Z in the MSSM and NHSSM in a comparative fashion.

2.1 Evolution of soft terms

For the soft breaking parameters of the NHSSM [8], we use one-loop Renormalization Group Equations (RGEs) [11] and thereby express their weak scale values in terms of GUT boundary conditions (see Appendix A). Once weak scale mass values of SUSY particles are known, it will be possible to make educated guesses as to the GUT side. Meanwhile, the most general semi-analytic solution set of the RGEs for the NHSSM is too large for practical purposes to carry out phenomenological analyses which we present in Appendix B. Nevertheless, the number of free parameters can be considerably reduced if one assumes the universality of the soft terms at the GUT scale. In this case solutions are phenomenologically more viable and they can be found in Appendix C for all soft terms. Our choice for the GUT scale universality condition can be stated (dropping the contributions of all fermion generations but the third family) as some prototype structure inspired from minimal supergravity:

$$m_{H_u,H_d,t_L,t_R,b_R,l_L,l_R}(0) \to m_0\,,\; \mu'(0) \to \mu'_0\,,$$
$$A_{t,b,\tau}(0) \to A_0\,,\; A'_{t,b,\tau}(0) \to A'_0\,,\; M_{1,2,3}(0) \to M\,. \tag{4}$$

Clearly, one may relax all or part of these conditions whereby obatining a larger parameter space augmenting the results presented in Appendix B. One should note that even if universal soft masses are assumed at the Planck scale, consideration of different boundary conditions for all soft terms including phases is more elegant, but then it gets difficult to achieve cetain clear-cut statements from the phenomenological side. To evade this cumbersome reality one needs certain inspirations which can be expected from string models. In order to use the most general one-loop solutions presented in this work, one can choose for instance, if the initial value of gauginos are not necessarily the same, then $M_{30} \neq M_{20} \neq M_{10}$ can be implemented, and this approach can be generalized to all soft breaking terms.

One of the most important distinctions is that, in the MSSM none of the soft masses depend on the initial value of μ, whereas in NHSSM both A' parameters and soft masses do depend on μ'_0. Using the universality conditions of (4), let us present some of the soft masses in both of the models for low $tan\beta$ choice ($tan\beta$=5). In the MSSM masses of up and down Higgs at the weak scale can be expressed using boundary conditions of common gaugino mass, cubic and soft mass squared terms,

$$m^2_{H_u}(t_Z) = -0.087A_0^2 + 0.38A_0M - 0.16m_0^2 - 2.8M^2,$$
$$m^2_{H_d}(t_Z) = -0.0033A_0^2 + 0.011A_0M + 0.99m_0^2 + 0.49M^2, \tag{5}$$

whereas in the NHSSM also have primed trilinear couplings,

$$m^2_{H_u}(t_Z) = -0.087A_0^2 + 0.1A_0'^2 - 0.16m_0^2 - 2.8M^2 + 0.067A'_0\mu'_0 + 0.14\mu_0'^2 + 0.38A_0M, \tag{6}$$
$$m^2_{H_d}(t_Z) = -0.0033A_0^2 - 0.37A_0'^2 + 0.99m_0^2 + 0.49M^2 - 0.31A'_0\mu'_0 + 0.6\mu_0'^2 + 0.011A_0M.$$

As it is seen in (5,6), at the electroweak scale, the results are the same except primed trilinear couplings and μ_0, μ'_0 terms. As a matter of fact NHSSM predictions reduces to that of MSSM results under the following transformation:

$$\mu', A'_t, A'_b, A'_\tau \to \mu\,,\; m^2_{H_{u,d}} \to m^2_{H_{u,d}} + \mu^2, \tag{7}$$

which declares that NHSSM is a beautiful extension of the MSSM. In the NHSSM, notice that the contribution of $A_0'^2$ terms is not of the same order of A_0^2 terms for all soft masses, hence trilinear and primed-trilinear couplings are not symmetric (see Appendix C). What is more interesting is that, for both of the models, all soft masses depend heavily on the gaugino masses with the exception of leptons $m^2_{l_{L,R}}$. Among others $m^2_{t_L}$ is the most sensitive not only for gaugino masses but also for the initial value of μ', for the latter $m^2_{H_d}$ is the least sensitive in the NHSSM.

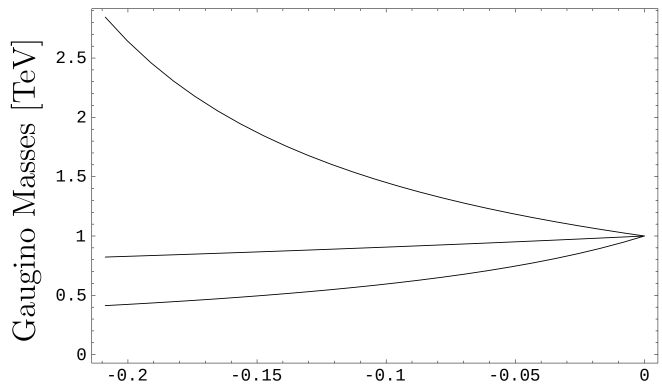

Figure 1: Scale dependency of gauginos in both of the models. Notice that here the boundary value of M is assumed to be 1 TeV. Scale dependency is expressed by dimensionless t such that t_0 corresponds to 1.9×10^{16} GeV . Here, Bino is at the bottom, followed by Wino and Gluino. Note that the same figure shows unification of gauge couplings.

At this point it is appropriate to stress that there are also common model independent predictions like the evolution of gauiginos (i.e. see Fig.1), which stems from the insensitiveness of gauge and Yukawa RGEs to both of the models at one-loop. On the other hand, trilinear couplings and other soft terms can be seen, in a way, to transformed into a new set in which μ terms are replaced with primed terms.

2.2 M_Z boundary

For both of the models, as one of the most crucial constraints for the SM agreement with data, mass of the Z boson should be considered first, for a successful electroweak symmetry breaking. Notice that in the MSSM, in order to get the observed value of M_Z, a delicate cancellation between the Higgs masses and μ is required, which is the famous μ problem (see i.e. [12],[5]). Instead of μ parameter of the MSSM, NHSSM bears $A_{t'}$, $A_{b'}$, $A_{\tau'}$ and μ' and its interesting effect can be seen by minimizing the scalar potential of the NHSSM which brings the constraint

$$\frac{M_Z^2(t_Z)}{2} = \frac{m^2_{H_d}(t_Z) - tan^2\beta\, m^2_{H_u}(t_Z)}{tan^2\beta - 1}. \tag{8}$$

The Z boson mass depends on μ_0 rather strongly in the MSSM. As an example for $tan\beta$=5, MSSM constraints can be expressed under the assumption of universality as

$$\frac{M_Z^2(t_Z)}{2} = 0.09A_0^2 + 0.21m_0^2 + 3M^2 - 0.92\mu_0^2 - 0.39A_0M. \tag{9}$$

However, in NHSSM it does depend on μ' rather weakly *e.g.* a 10% change in $\mu_0'^2$ generates only a 0.1% shift in $M_Z^2/2$. To make a comparison, in the NHSSM for the same value of $tan\beta$:

$$\frac{M_Z^2(t_Z)}{2} = 0.09A_0^2 - 0.12A_0'^2 + 0.21m_0^2 + 3M^2 - 0.082A_0'\mu_0' - 0.12\mu_0'^2 - 0.39A_0M. \quad (10)$$

For the sake of visualization of the NHSSM and MSSM reactions we define dimensionless quantities $\gamma_i(tan\beta)$ such that the Z constrain can be expressed as

$$\frac{M_Z^2(t_Z)}{2} = \gamma_1'A_0^2 + \gamma_2'A_0'^2 + \gamma_3'm_0^2 + \gamma_4'M^2 + \gamma_5'A_0'\mu_0' + \gamma_6'\mu_0'^2 + \gamma_7'A_0M, \quad (11)$$

which can be used also for MSSM with obvious modifications. In the range $tan\beta$ ϵ [2,60], weights of γ's can be inferred from Figs.2,3 and 4.

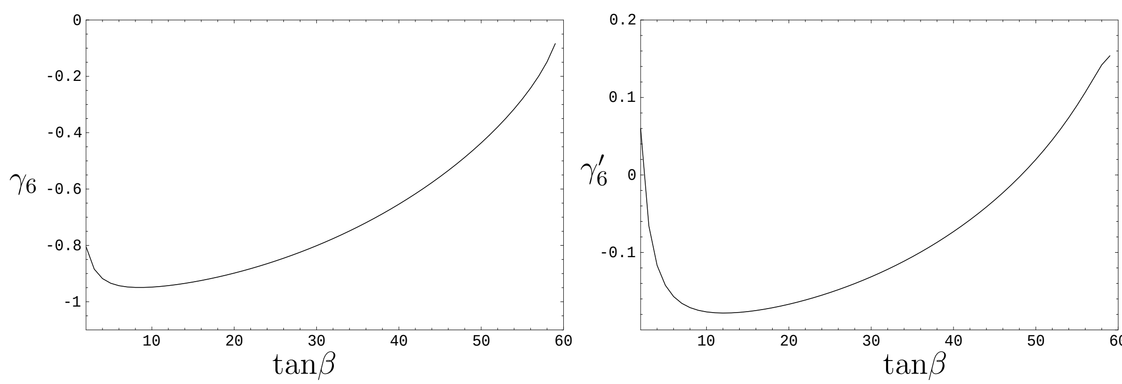

Figure 2: Evolution of the coefficients of μ^2 terms versus $tan\beta$ ϵ [2,60] in the MSSM (left), and of μ'^2 terms in the NHSSM (right) satisfying M_Z constraint.

In addition to relaxing sensitivity on the μ_0 terms, we observe that $tan\beta$ changes the sign of the μ' contribution in the NHSSM, and this situation has important consequences on the model building business. Note that in the MSSM contribution of μ^2 terms is always destructive (assuming it is real), whereas by staring the oscillatory behaviour of μ'^2 with different choices of $tan\beta$ (see Fig.2) one can find a specific prediction for $tan\beta$ such that μ'^2 dependency of the M_Z^2 completely vanishes in low and high regions, in addition to destructive or constructive contribution regions. Such special points can be called as turning points and this corresponds to ~ 49.25 for high $tan\beta$ in the NHSSM under the assumption of universal terms. Of course relaxing the universality assumption brings different turning points.

In addition to capability of getting rid off μ' terms for specific angles, NHSSM deserves new phenomenological approach which can be inferred from the Figs.2,3 and 4. In the figures $tan\beta$ evolution of the coefficients of mass dimension 2 terms satisfying the M_Z constraint is given. Behavior of other terms should also be taken into account, which can be performed using the Fig.4 to unfold the reaction of the model satisfying the mass boundary of the Z boson. To make a comparison with the MSSM we also presented the $tan\beta$ evolution of similar coefficients in Fig.3. Using figures presented here, one can find appropriate regions that

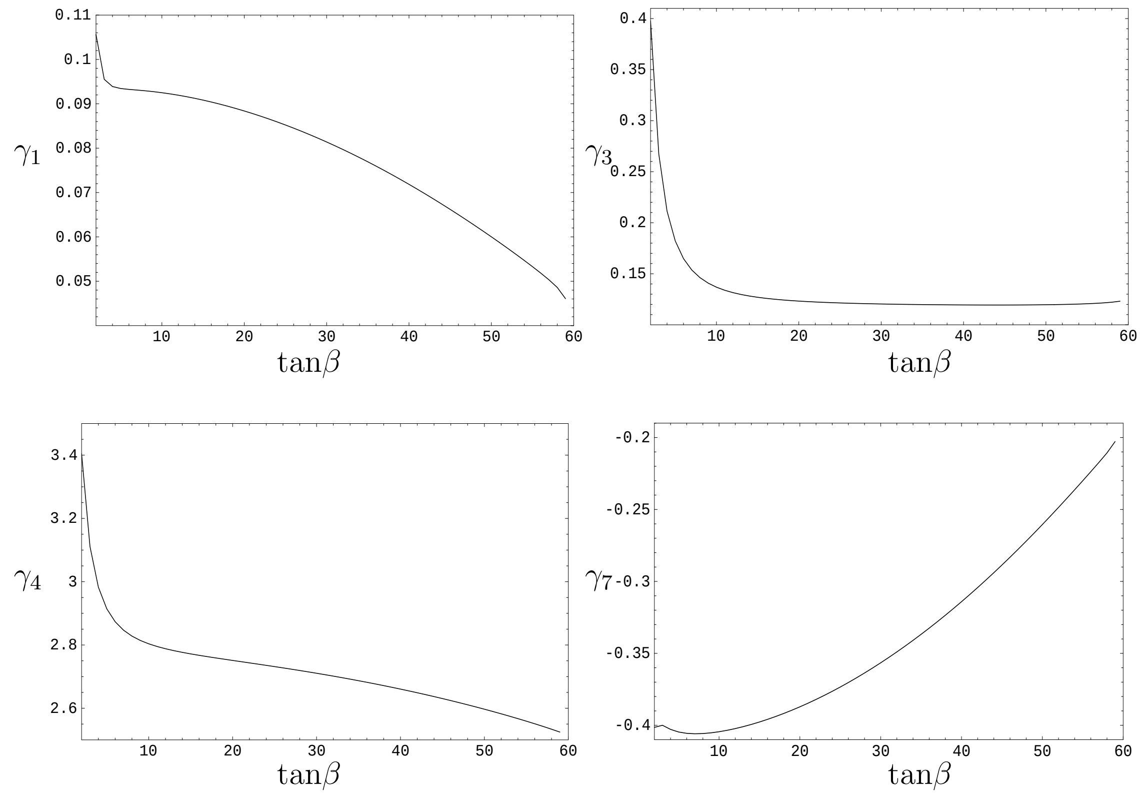

Figure 3: Evolution of the $\gamma_{1,3,4,7}$ terms versus $tan\beta$ ϵ [2,60] in the MSSM

satisfy the Z constrain in both of the models for any $tan\beta$ in the range [2,60]. Nevertheless, one should notice that constraints presented here are at the tree level and in the universal region, which might change when radiative corrections or anomaly boundary conditions are considered.

Consequently, supersymmetry breaking with non-standard soft terms has an important virtue of reducing the sensitivity of M_Z^2 to the initial value of the μ parameter. However, in both cases, the MSSM and NHSSM, the Z boson mass exhibits a strong sensitivity of the gaugino masses. This follows mainly from the asymptotic freedom of color gauge group.

3 Spectrum of sparticles in Minimal Supergravity: MSSM vs. NHSSM

From the viewpoint of realistic model building approach any model should satisfy other collider bounds besides M_Z, however we know from direct searches that no supersymmetric particle is observed yet, which can not set tight bounds on the spectrum of masses of SUSY particles [16]. Meanwhile mass of Higgs boson can be considered as on the verge of experimental verification if low scale supersymmetry really exists. We consider particle data group restrictions on the mass of sparticles and simply accept the lower bounds of LEP 2 $m_{soft} > 100$ GeV, for the lightest chargino and neutralino half of Z boson width is accepted

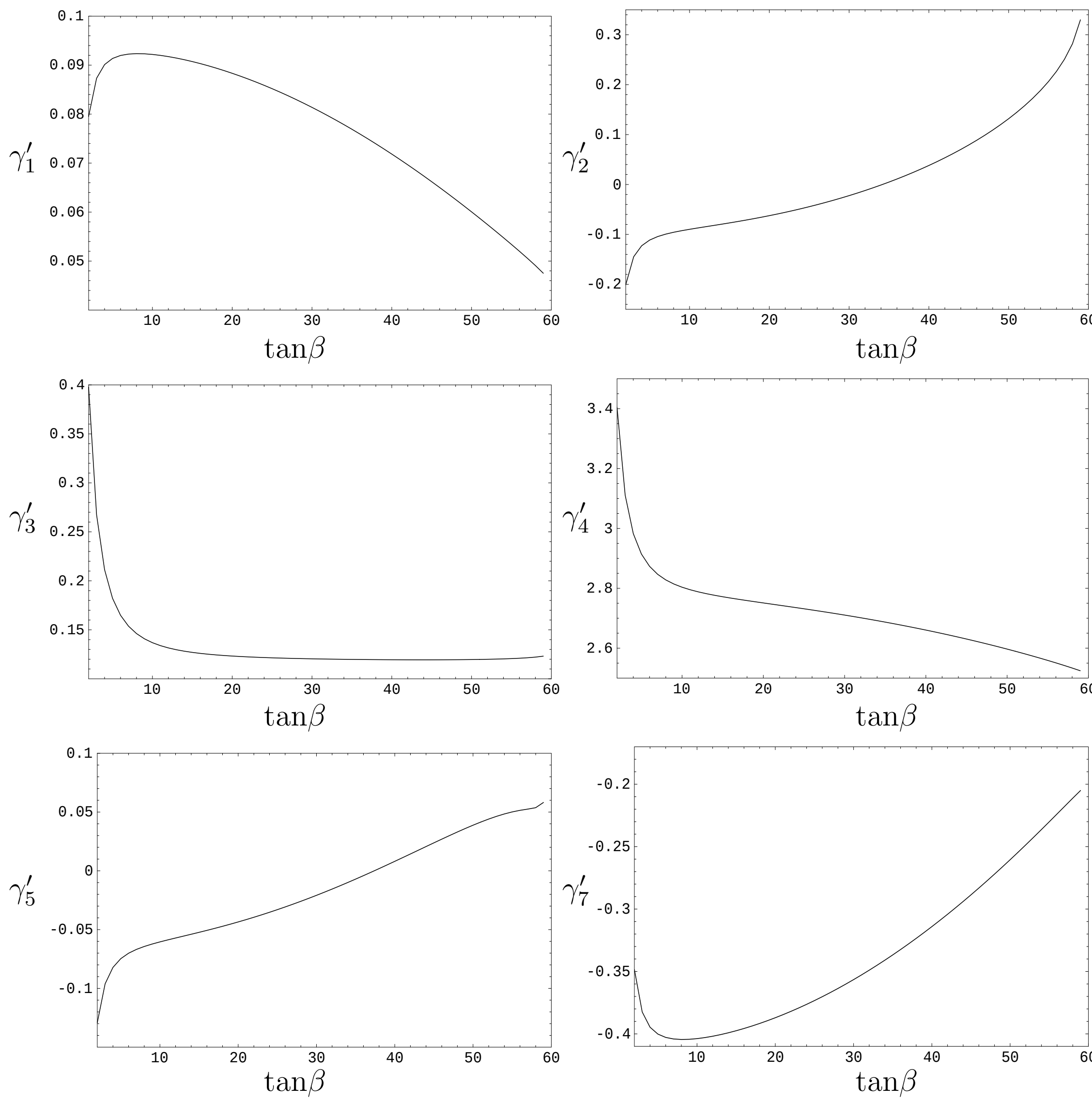

Figure 4: Evolution of the $\gamma'_{1,2,3,4,5,7}$ terms versus $tan\beta \; \epsilon \; [2,60]$ in the NHSSM

[17]. For simplicity and clarity, again, in this section we require all scalars to acquire a common mass m_0, all gauginos to be mass-degerate with M, all triscalar couplings to be A_0 and all non-holomorphic triscalars to be A'_0 all fixed at the GUT scale. In fact, suppression of the flavor-changing neutral currents as well as the absence of permanent electric dipole moments already imply that the soft-breaking masses cannot be all independent and arbitrarily distributed; they must be correlated by some organizing principle operating at the unification scale or above. With this assumption one can predict mass of lightest Higgs boson at tree level using the scalar Higgs potential of the NHSSM which brings the constraints

$$m^2_{H_d} = m^2_3 \, tan\beta - (M^2_Z/2) \, cos2\beta, \tag{12}$$

$$m^2_{H_u} = m^2_3 \, cot\beta + (M^2_Z/2) \, cos2\beta. \tag{13}$$

During the numerical investigation, we look for real and positive soft terms in the range $[0, 1000]$ GeV, which results in successful electroweak symmetry breaking patterns for low $tan\beta$ option. In this case by noting the collider lower bounds on the mass spectrum, parameter space can be restricted to a good extend, without additional assumptions (like no-scale [18], or some other string inspired models). With the same range proposed for GUT boundaries there is no succesfull candidate in high $tan\beta$ region, while the universality assumption of (4) in charge. When the electroweak symmetry is broken mass eigenstate of the lightest neutral scalar should satisfy $m_h^0 > 114$ GeV with radiative corrections. By expanding the scalar potential around the minimum tree-level masses of the fields can be found as

$$m_{A^0}^2 = 2m_3^2/sin2\beta, \tag{14}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + M_W^2, \tag{15}$$

$$m_{h^0,H^0}^2 = \frac{1}{2}(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4M_Z^2 m_{A^0}^2 cos^2 2\beta}), \tag{16}$$

when one-loop quantum corrections are considered SM like Higgs boson gets the largest contributions from t and b squarks. Notice that without quantum corrections mass of the lightest Higgs boson can not satisfy the experimental boundary, hence we study this issue in section 3.3 for NHSSM without CP violation; MSSM results including CP violation can be found in [19, 20] Analytic forms of $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ is given in the following subsection which will be needed in correction business.

3.1 Sfermions

For scalar fermions the relation between gauge eigenvalues and mass eigenvalues of the NHSSM particles can be read from the mass-squared matrices. Following that aim, we provide explicit expressions for the mass-squared matrices of squark and sleptons using reference [10]. The stop matrix is:

$$\begin{pmatrix} m_{t_L}^2 + m_t^2 + \frac{1}{6}(4M_W^2 - M_Z^2)\cos 2\beta & m_t(A_t - A_{t'}\cot\beta) \\ m_t(A_t - A_{t'}\cot\beta) & m_{t_R}^2 + m_t^2 - \frac{2}{3}(M_W^2 - M_Z^2)\cos 2\beta \end{pmatrix}. \tag{17}$$

for which we obtain the following eigenvalues

$$\begin{aligned} m_{\tilde{t}_{1,2}}^2 &= \frac{1}{12}\{6(2m_t^2 + m_{tL}^2 + m_{tR}^2) + 3M_Z^2 cos2\beta \\ &\mp \sqrt{\sigma_1 cos2\beta\,(12\sigma_2 + \sigma_1 cos2\beta) + 36\,[4A_t^2 m_t^2 + \sigma_2^2 + 4A_{t'} m_t^2 cot\beta\,(-2A_t + A_t' cot\beta)]}\}, \end{aligned} \tag{18}$$

where $\sigma_1 = 8M_W^2 - 5M_Z^2$ and $\sigma_2 = m_{tL}^2 - m_{tR}^2$. Similarly for the bottom squarks we have:

$$\begin{pmatrix} m_{t_L}^2 + m_b^2 - \frac{1}{6}(2M_W^2 + M_Z^2)\cos 2\beta & m_b(A_b - A_{b'}\tan\beta) \\ m_b(A_b - A_{b'}\tan\beta) & m_{b_R}^2 + m_b^2 + \frac{1}{3}(M_W^2 - M_Z^2)\cos 2\beta \end{pmatrix} \tag{19}$$

with eigenvalues

$$\begin{aligned} m_{\tilde{b}_{1,2}}^2 &= \frac{1}{12}\{6(2m_b^2 + m_{tL}^2 + m_{bR}^2) - 3M_Z^2 cos2\beta \\ &\mp\sqrt{\sigma_3 cos2\beta(12\sigma_4 - \sigma_3 cos2\beta) + 36\,[4A_b^2 m_b^2 + \sigma_4^2 + 4A_{b'} m_b^2 tan\beta(-2A_b + A_b' tan\beta)]}\}, \end{aligned} \tag{20}$$

where $\sigma_3 = 4M_W^2 - M_Z^2$ and $\sigma_4 = m_{\tilde{b}R}^2 - m_{\tilde{t}L}^2$. For the tau sleptons we have:

$$\begin{pmatrix} m_{\tilde{l}_L}^2 + m_\tau^2 - \frac{1}{2}(2M_W^2 - M_Z^2)\cos 2\beta & m_\tau(A_\tau - A'_\tau \tan\beta) \\ m_\tau(A_\tau - A'_\tau \tan\beta) & m_{\tilde{l}_R}^2 + m_\tau^2 + (M_W^2 - M_Z^2)\cos 2\beta \end{pmatrix}. \tag{21}$$

for which eigenvalues can be written as

$$\begin{aligned} m_{\tilde{\tau}_{1,2}}^2 &= \frac{1}{4}\{2(2m_b^2 + m_{\tilde{l}L}^2 + m_{\tilde{l}R}^2) - M_Z^2 cos2\beta \\ &\mp\sqrt{\sigma_5 cos2\beta(\sigma_5 - 4\sigma_6 cos2\beta) + 4\left[4A_\tau^2 m_\tau^2 + \sigma_6^2 + 4A'_\tau m_\tau^2 tan\beta(-2A_\tau + A'_\tau tan\beta)\right]}\}, \end{aligned} \tag{22}$$

where $\sigma_5 = 4M_W^2 - 3M_Z^2$ and $\sigma_6 = m_{\tilde{l}L}^2 - m_{\tilde{l}R}^2$. Explicit expressions related with each of the elements of these matrices can be extracted from the Appendix C of this work for low and high $tan\beta$ choices. In the MSSM sfermion masses depend on μ_0 only via their (1,2) and (2,1) entires whereas in the NHSSM μ'_0 appears in all entires including (1,1) and (2,2). When all the Yukawa couplings are set to zero, except h_t and h_τ, it is interesting to observe SUSY loop effects on the mass squared terms (see [26] and [27]).

3.2 Charginos and Neutralinos

The last step is to compare the mass eigenvalues of neutralinos and charginos. Neutralino values can be read from the following matrix, which resembles the mixing of Higgsinos and neutral gauginos

$$\begin{pmatrix} M_1 & 0 & -M_Z\, cos\beta\, sin\theta_W & M_Z\, sin\beta\, sin\theta_W \\ 0 & M_2 & M_Z\, cos\beta\, cos\theta_W & -M_Z\, sin\beta\, cos\theta_W \\ -M_Z\, cos\beta\, sin\theta_W & M_Z\, cos\beta\, cos\theta_W & 0 & -\mu' \\ M_Z\, sin\beta\, sin\theta_W & -M_Z\, sin\beta\, cos\theta_W & -\mu' & 0 \end{pmatrix}. \tag{23}$$

Similarly charginos are mixtures of charged Higgsinos and charged gauginos with the mass matrix

$$\begin{pmatrix} M_2 & \sqrt{2}M_W sin\beta \\ \sqrt{2}M_W sin\beta & \mu' \end{pmatrix}. \tag{24}$$

Since we assume R-parity conservation LSP is the lightest neutralino. Explicit form of matrix elements can be found in Appendices for low and high values of $tan\beta$.

3.3 Higgs boson mass and LEP bounds

In this section we will compute the Higgs boson mass in NHMSSM. The main impact of the non-holomorphic soft terms on the Higgs boson masses stems from the modifications in the sfermion mass matrices. Indeed, as one infers from the forms of the sfermion mass-squared matrices in Sec. 3.2, the mixing between the left and right-handed sfermions are described by the holomorphic triscalar coupling A_t and the non-holomorphic contribution A'_f. The left-right mixing thus changes from flavor to flavor in contrast to MSSM where A'_f is replaced by flavor-insensitive quantity μ parameter.

For a proper understanding of the Higgs sector it is necessary to implement the loop corrections as otherwise the tree level masses turn out to be too low to saturate the experimental bounds. The radiative corrections to Higgs boson masses and couplings have already been computed in [19, 20] including the CP-violating effects. Concerning the neutral Higgs sector, it is useful to use the parametrization

$$H_d^0 = \frac{1}{\sqrt{2}}\left(\phi_1 + i\varphi_1\right);\ H_u^0 = \frac{1}{\sqrt{2}}\left(\phi_2 + i\varphi_2\right), \tag{25}$$

where $\phi_{1,2}$ and $\varphi_{1,2}$ are real fields. The Higgs potential, including the Coleman-Weinberg contribution, reads as

$$\begin{aligned} V_{\text{Higgs}} &= \frac{1}{2}m_{H_d}^2|H_d^0|^2 + \frac{1}{2}m_{H_u}^2|H_u^0|^2 - (m_3^2 H_u^0 H_d^0 + c.c.) \\ &+\frac{g^2+g'^2}{8}\left(|H_d^0|^2 - |H_u^0|^2\right)^2 + \frac{1}{64\pi^2}\text{Str}\left[\mathcal{M}^4\left(\log\frac{\mathcal{M}^2}{Q_0^2} - \frac{3}{2}\right)\right], \end{aligned} \tag{26}$$

where g and g' stand for the $SU(2)$ and $U(1)_Y$ gauge couplings, respectively ($g'^2 = \frac{3}{5}g_1^2$). Q_0 in (26) is the renormalization scale, and $\mathcal{M}$ is the field–dependent mass matrix of all modes that couple to the Higgs bosons. The masses of the quarks are to be taken into consideration of which the most important contributions come from:

$$m_b^2 = \frac{1}{2}h_b^2\left(\phi_1^2 + \varphi_1^2\right); \quad m_t^2 = \frac{1}{2}h_t^2\left(\phi_2^2 + \varphi_2^2\right). \tag{27}$$

Now, using the eigenvalues of the field-dependent squark mass matrices (18,20) in (26) one can systematically compute the Higgs boson masses at the minimum of the potential obtained via the conditions

$$\frac{\partial V_{Higgs}}{\partial \phi_1} = 0,\ \frac{\partial V_{Higgs}}{\partial \phi_2} = 0 \tag{28}$$

with $\langle\varphi_1\rangle = \langle\varphi_2\rangle = 0$ and

$$\langle\phi_1\rangle^2 + \langle\phi_2\rangle^2 = \frac{M_Z^2}{\hat{g}^2} \simeq (246\text{ GeV})^2,\ \frac{\langle\phi_2\rangle}{\langle\phi_1\rangle} = \tan\beta, \tag{29}$$

where $\hat{g}^2 = (g^2 + g'^2)/4$. The mass matrix of the neutral Higgs bosons are computed from the matrix of second derivatives of the potential (26). Notice that after including the one-loop corrections to the Higgs potential, the Z mass becomes dependent on the top- and stop quark masses too [29]. In this case there will be a correction term

$$\frac{M_Z^2(t_Z)}{2} = \frac{m_{H_d}^2(t_Z) - tan^2\beta\, m_{H_u}^2(t_Z) - \Delta_Z^2(t,b)}{tan^2\beta - 1}. \tag{30}$$

where

$$\Delta_Z^2(t) = \frac{3g^2 m_t^2}{32\pi^2 M_W^2}\left[\left(A_t^2 - A_{t'}^2 cot^2\beta\right)\frac{f(m_{\tilde{t}_1}^2) - f(m_{\tilde{t}_2}^2)}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} + 2m_t^2 + f(m_{\tilde{t}_1}^2) + f(m_{\tilde{t}_2}^2)\right] \tag{31}$$

and

$$f(m^2) = 2m^2 \left(\log \frac{m^2}{Q_0^2} - 1 \right). \tag{32}$$

Similarly $\Delta_Z^2(b)$ can be found with the $t \to b$ substitution. This corrections require a large amount of fine tuning if the mass splitting between the particles and sparticles is large [8].

The Goldstone boson $G^0 = \varphi_1 \cos\beta - \varphi_2 \sin\beta$ is swallowed by the Z boson. We are then left with a squared mass matrix $\mathcal{M}_H^2$ for the three states $\varphi = \varphi_1 \sin\beta + \varphi_2 \cos\beta$, ϕ_1 and ϕ_2. If the theory has CP-violating phases (via the phases of the triscalar couplings and μ') the φ mixes with ϕ_1 and ϕ_2. In the CP-conserving limit, however, φ decouples from the rest, and assumes the mass-squared:

$$\mathcal{M}_H^2\Big|_{aa} = m_A^2 = \frac{2m_3^2}{\sin(2\beta)} + \frac{2}{\sin(2\beta)} \left[h_t^2 A_t A_{t'} F(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + h_b^2 A_b A_{b'} F(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \right], \tag{33}$$

where

$$F(m_1^2, m_2^2) = \frac{3}{32\pi^2} \frac{f(m_1^2) - f(m_2^2)}{m_2^2 - m_1^2}. \tag{34}$$

The remaining real scalars ϕ_1 and ϕ_2 mix with each other via the mass-sqaured matrix:

$$\begin{aligned}
\mathcal{M}_H^2\Big|_{\phi_1\phi_1} &= M_Z^2 \cos^2\beta + m_A^2 \sin^2\beta \\
&+ \frac{3m_t^2}{8\pi^2} \left[g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) R_t \left(h_t^2 R_t - \cot\beta X_t \right) + \hat{g}^2 \cot\beta R_t \log \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right] \\
&+ \frac{3m_b^2}{8\pi^2} \left\{ h_b^2 \log \frac{m_{\tilde{b}_1}^2 m_{\tilde{b}_2}^2}{m_b^4} - \hat{g}^2 \log \frac{m_{\tilde{b}_1}^2 m_{\tilde{b}_2}^2}{Q_0^4} \right. \\
&\left. + g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) R_b' \left(h_b^2 R_b' + X_b \right) + \log \frac{m_{\tilde{b}_2}^2}{m_{\tilde{b}_1}^2} \left[X_b + \left(2h_b^2 - \hat{g}^2 \right) R_b' \right] \right\};
\end{aligned} \tag{35}$$

$$\begin{aligned}
\mathcal{M}_H^2\Big|_{\phi_2\phi_2} &= M_Z^2 \sin^2\beta + m_A^2 \cos^2\beta \\
&+ \frac{3m_t^2}{8\pi^2} \left\{ h_t^2 \log \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} - \hat{g}^2 \log \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{Q_0^4} \right. \\
&\left. + g(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) R_t' \left(h_t^2 R_t' + X_t \right) + \log \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \left[X_t + \left(2h_t^2 - \hat{g}^2 \right) R_t' \right] \right\} \\
&+ \frac{3m_b^2}{8\pi^2} \left[g(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) R_b \left(h_b^2 R_b - \tan\beta X_b \right) + \hat{g}^2 \tan\beta R_b \log \frac{m_{\tilde{b}_2}^2}{m_{\tilde{b}_1}^2} \right].
\end{aligned} \tag{36}$$

where

$$g(m_1^2, m_2^2) = 2 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \log \frac{m_1^2}{m_2^2}, \tag{37}$$

and

$$X_t = \frac{5g'^2 - 3g^2}{12} \cdot \frac{m_{t_L}^2 - m_{t_R}^2}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \; ; X_b = \frac{g'^2 - 3g^2}{12} \cdot \frac{m_{t_L}^2 - m_{b_R}^2}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2}, \tag{38}$$

$$R_t = \frac{A_{t'}^2 \cot\beta + A_t A_{t'}}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}\ ; R_t' = \frac{A_t^2 + A_t A_{t'} \cot\beta}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \tag{39}$$

$$R_b = \frac{A_{b'}^2 \tan\beta + A_b A_{b'}}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2}\ ; R_b' = \frac{A_b^2 + A_b A_{b'} \tan\beta}{m_{\tilde{b}_2}^2 - m_{\tilde{b}_1}^2}\,. \tag{40}$$

It is known that, the two-loop corrections to Higgs boson mass are reduced at the renormalization scale $Q_0 = m_t$ hence our choice hereon. To give a concrete example of NHSSM

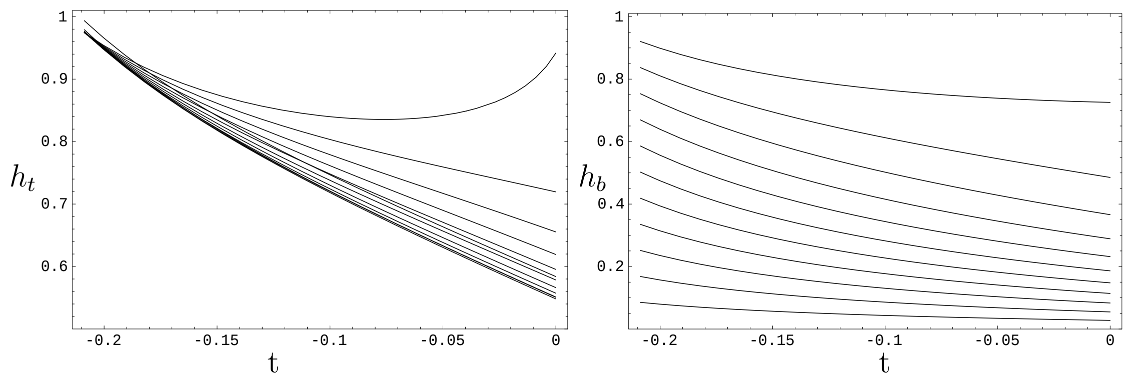

Figure 5: Scale dependence of the couplings h_t (left) and of h_b (right) for different choices of $tan\beta$ ϵ [5,55]. In both of the figures topmost curves correspond to $tan\beta$=55.

benchmark we now list mass predictions of the model for low $tan\beta$ with the input parameters; $\bar{m}_t(t_Z) = 170$, $\bar{m}_b(t_Z) = 2.92$ and $\bar{m}_\tau(t_Z) = 1.777$ GeV and take the GUT boundary values of soft terms as the following set

$$M = 160,\ m_0 = 683,\ \mu_0' = 400,\ A_0 = 800,\ A_0' = 1000,\ m_{30} = 430 \tag{41}$$

which brings the following predictions

$$\begin{aligned}
&m_{\tilde{t}_1}(t_Z) = 291,\ m_{\tilde{t}_2}(t_Z) = 626,\\
&m_{\tilde{b}_1}(t_Z) = 600,\ m_{\tilde{b}_2}(t_Z) = 791,\\
&m_{\tau_1}(t_Z) = 683,\ m_{\tau_2}(t_Z) = 695,\\
&m_{\chi^0_{1,2,3,4}}(t_Z) = 63,\ 120,\ 392,\ 407,\\
&m_{\chi^\pm_{1,2}}(t_Z) = 119\,, 407,\\
&m_{A^0}(t_Z) = 289,\ m_{H^\pm}(t_Z) = 300,\\
&m_{H^0}(t_Z) = 291,\ m_{h^0}(t_Z)_{corrected} = 123,
\end{aligned} \tag{42}$$

where all masses are given in GeV.

Since NHSSM covers MSSM any prediction of the classical MSSM results can be reproduced in non-holomorphic case with the appropriate boundaries. But the extension enriches us with more opportunities. What it is important here is the degree of freedom offered by NHSSM. As it was stressed in [10] for $m_0 \gg M$ it turns out that $|\mu| < 0.4\ M$ in the MSSM

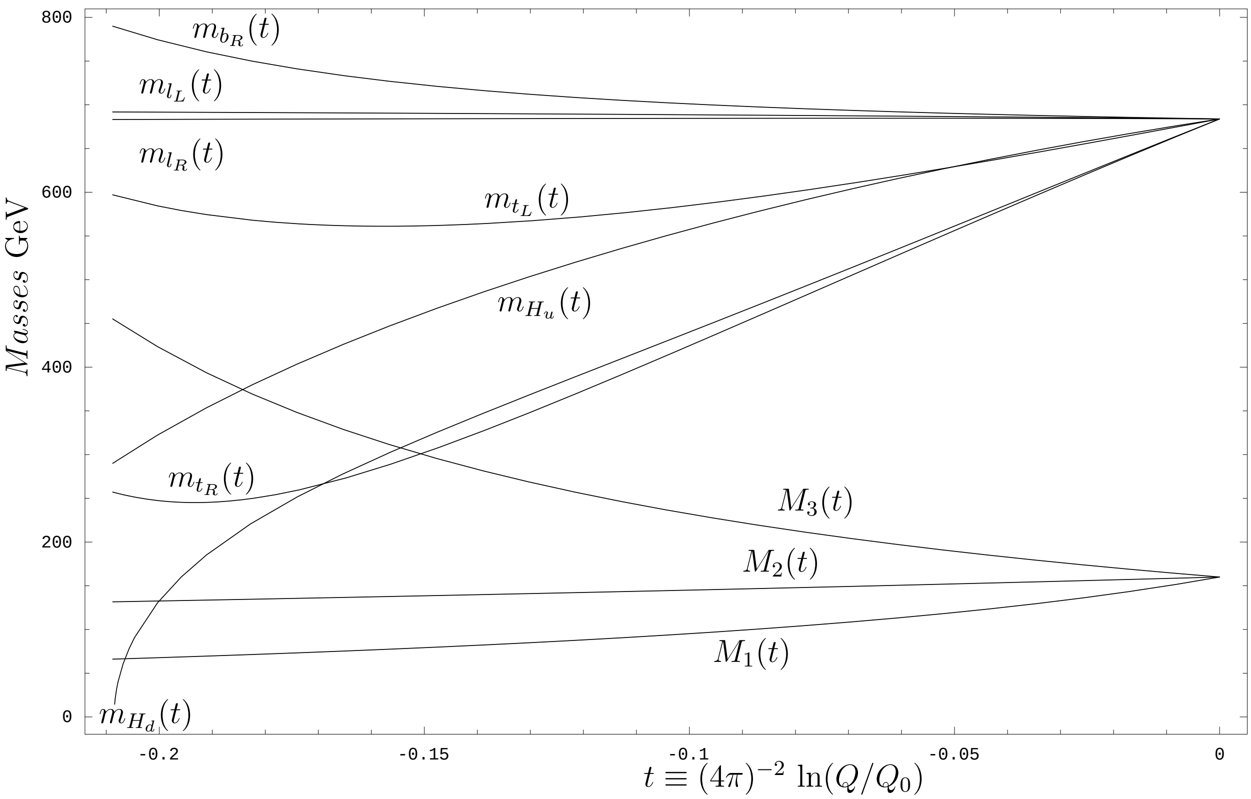

Figure 6: A sample plot of some of the soft terms versus scale in the NHSSM with the input parameters given in the text.

whereas in the NHSSM this constrained is significantly relaxed. Note that in our example we assumed all soft terms as if they are real and positive without considering any specific model, whereas one can study i.e $A_0 = -M$ which arises in certain string inspired models. Under the light of these observations, it should be stated that, NH extension of the MSSM not only covers the classical MSSM but also offers novel features that can ease the shortcomings of the MSSM, which should be studied in more detail. Actually, in addition to LEP limits on the SUSY mass spectrum, one should also deal with the constraints from $b \to s\gamma$ decay (as we do in next subsection) and the lower limit on the lifetime of the universe, which requires the dark matter density from the LSP not to close the universe on itself [30].

3.4 $b \to s\gamma$ Decay

Presently, one of the most accurate observables which can severely constrain the soft masses is the branching ratio for the rare radiative inclusive B meson decay, $B \to X_s\gamma$. The main interest in this decay drives from the genuine perturbative nature of the problem and also from the striking agreement between the experiment and the SM prediction. Indeed, the measurements of the branching ratio at CLEO, ALEPH and BELLE gave the combined result [21]

$$\mathrm{BR}\,(B \to X_s\gamma) = (3.11 \pm 0.42 \pm 0.21) \times 10^{-4} \tag{43}$$

whose agreement with the next–to–leading order (NLO) standard model (SM) prediction [22]

$$\mathrm{BR}\,(B \to X_s\gamma)_{\mathrm{SM}} = (3.29 \pm 0.33) \times 10^{-4} \tag{44}$$

is manifest though the inclusion of the nonperturbative effects can modify the result slightly [23]. That the experimental result (43) and the SM prediction (44) are in good agreement shows that the "new physics" should lie well above the electroweak scale unless certain cancellations occur.

The branching ratio for $B \to X_s\gamma$ has been computed up to NLO precision in the MSSM [24]. The W boson and charged Higgs contributions are of the same sign and thus the chargino–stop loop is expected to moderate the branching ratio so as to respect the experimental bounds. The recent measurements of $\mathrm{BR}\,(B \to X_s\ell^+\ell^-)$ [25] imply that the sign of the total $b \to s\gamma$ amplitude must be same as in the SM. This eliminates part of the supersymmetric parameter space in which the total amplitude approximately equals negative of the SM prediction. In spite of these, however, the present experimental results do not exclude stop masses around a few M_Z as long as A_t and A'_t are of opposite sign [24].

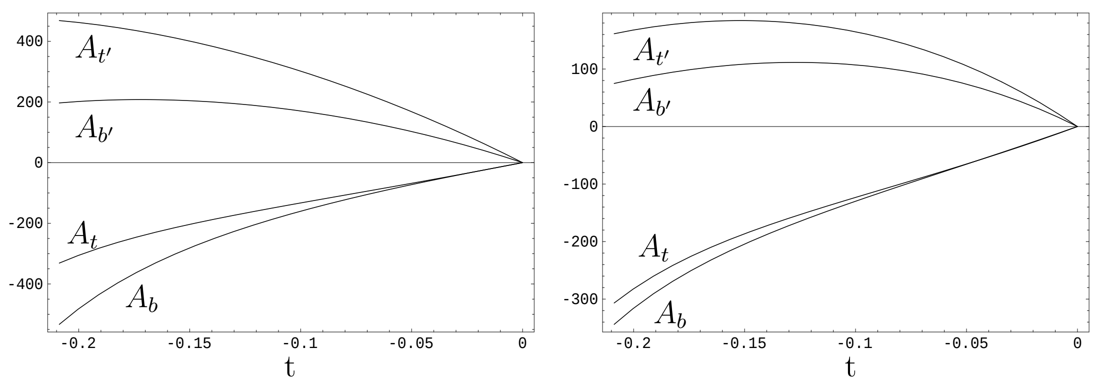

Figure 7: A sample plot of the scale dependence of the trilinear couplings for $tan\beta$=5 (left), $tan\beta$=50 (right) with $A_0 = A'_0$=0, M=150 GeV and μ'=1000 GeV, which show a candidate region where A_t and A'_t are of opposite sign.

To accommodate differing signs of trilinear couplings in the NHSSM we present another example using the following input parameter

$$M = 200,\, m_0 = 787,\, \mu'_0 = 400,\, A_0 = 900,\, A'_0 = -1500,\, m_{30} = 414 \tag{45}$$

which yields the following predictions

$$m_{\tilde{t}_1}(t_Z) = 362,\, m_{\tilde{t}_2}(t_Z) = 728,$$

$$
\begin{aligned}
&m_{\tilde{b}_1}(t_Z) = 711,\, m_{\tilde{b}_2}(t_Z) = 930, \\
&m_{\tau_1}(t_Z) = 787,\, m_{\tau_2}(t_Z) = 801, \\
&m_{\chi^0_{1,2,3,4}}(t_Z) = 79,\, 150,\, 392,\, 409 \\
&m_{\chi^\pm_{1,2}}(t_Z) = 149\,, 408, \\
&m_{A^0}(t_Z) = 299,\, m_{H^\pm}(t_Z) = 310, \\
&m_{H^0}(t_Z) = 301,\, m_{h^0}(t_Z)_{corrected} = 120,
\end{aligned}
\tag{46}
$$

here again all masses are given in GeV.

4 Renormalization Group Invariants in the MSSM and NHMSSM: A Comparative Analysis

Renormalization Group Invariants (RGIs), which can be used to relate measurements at the electroweak scale to physics at ultra high energies provide important information about high scale physics due to the scale invariance of the quantities under concern [31, 32]. Since the coupled nature of the RGEs disturbs analytical solutions it would be beneficial to know if one can construct certain invariants that give relations among the spectrum of supersymmetric particles. Indeed, RG invariants may provide a direct, accurate way of testing the internal consistency of the model and determine the mechanism which breaks the supersymmetry. Such quantities prove highly useful not only for projecting the experimental data to high energies but also for deriving certain sum rules which enable fast consistency checks of the model. Assume there is a measurement which tells a specific relation between some of the soft masses, then, it can be easily probed whether this relation survives at different scales or not, with the help of scale independent relations, which in turn shows the way how SUSY is broken.

In this part we will discuss RG invariant observables in supersymmetry with non-holomorphic soft terms and compare with existing MSSM results with the assumption that there is no flavor mixing and soft terms obey the universality condition mentioned previosly. Nevertheless, it should be kept in mind that we study one-loop RGIs which differs when R parity or higher loop effects are taken into account.

To begin with, note that lagrangian of the NHSSM (2) has parameters defined at a specific mass scale Q which can physically range from the electroweak scale $Q = M_Z$ (the IR end) up to some high energy scale $Q = Q_0$ (the UV end). For determining the scale dependencies of the parameters the RGEs are to be solved with proposed boundary conditions either at IR or UV. In what follows we will write them in terms of the dimensionless variable $t \equiv (4\pi)^{-2} \ln(Q/Q_0)$, and solve for the parameters in terms of their UV scale values by taking into account the fact that the gauge and Yukawa (at a given $\tan\beta$) couplings are already known at IR end.

We should deal with the rigid parameters in both of the models as a first step. The RGEs for gauge and Yukawa couplings form a coupled set of first order differential equations and can be found elsewhere (i.e. see [11]). Now one can solve them at any scale at one loop order without resorting to other model parameters. However, expanding this set of equations by including the RGE of the μ' parameter one finds that

$$I_1 = \mu' \left(\frac{g_2^9 \, g_3^{256/3}}{h_t^{27} \, h_b^{21} \, h_\tau^{10} \, g_1^{73/33}} \right)^{1/61} \tag{47}$$

is a one-loop RG-invariant. For the classical MSSM invariant $\mu' \to \mu$ substitution suffices (MSSM was also mentioned in [31]). Here the powers of the Yukawa and gauge couplings follow from group-theoretic factors appearing in their RGEs. This invariant provides an explicit solution for the μ' parameter

$$\mu'(t) = \mu'(0) \left(\frac{h_t(t)}{h_t(0)} \right)^{\frac{27}{61}} \left(\frac{h_b(t)}{h_b(0)} \right)^{\frac{21}{61}} \left(\frac{h_\tau(t)}{h_\tau(0)} \right)^{\frac{10}{61}} \left(\frac{g_3(0)}{g_3(t)} \right)^{\frac{256}{183}} \left(\frac{g_2(0)}{g_2(t)} \right)^{\frac{9}{61}} \left(\frac{g_1(t)}{g_1(0)} \right)^{\frac{73}{2013}} \tag{48}$$

once the scale dependencies of gauge and Yukawa couplings are known either via direct integration or via approximate solutions the RGE of the μ' parameter involves only the Yukawa couplings, g_2 and g_1 though this explicit solution bears an explicit dependence on g_3. This follows from the RGEs of the Yukawa couplings. One of the most interesting sides of this invariant is that weights of all gauge and Yukawa couplings is made obvious. With this equation one can determine the amount of fine tuning to satisfy Z mass boundary (see reference citedurmus for a detailed discussion on this issue). Another by-product of the invariant I_1 is that the phase of the μ parameter is an RG invariant. Since the contribution of higher order loop effects affect invariance relation of (47) $\sim 2-3\%$; an effect likely to get embodied in the experimental errors encourages us to work at one-loop order. On the other hand, once the flavor mixings in Yukawa matrices are switched on there is no obvious invariant like (47) even at one loop order.

We continue our analysis with the construction of the RG invariants of the soft parameters of the theory. Of this sector, a well-known RG invariant is the ratio of the gaugino masses to fine structure constants

$$I_2 = \frac{M_a}{g_a^2} \tag{49}$$

with one-loop accuracy. This very invariant guarantees that

$$M_a(t) = M_a(0) \left(\frac{g_a(t)}{g_a(0)} \right)^2 \tag{50}$$

so that knowing two of the gaugino masses at $Q = M_Z$ suffices to know the third – an important aspect to check directly the minimality of the gauge structure using the experimental data. Related with this invariant it is useful to state the well known mass ratios $M_3(t_Z)/M_2(t_Z) = 3.46$ and $M_2(t_Z)/M_1(t_Z) = 1.99$ at one-loop order. The invariant (49) pertains solely to the gauge sector of the theory; it is completely immune to non-gauge parameters. At two loops I_2 is no longer an invariant; it is determined by a linear combination of gaugino masses and trilinear couplings. Combining (48) and (50) one concludes that the chargino and neutralino sectors of the theory are connected to the UV scale via the gauge and Yukawa couplings alone. The equation (50) suggests that $M_3(t_Z)/M_3(0)$ is much larger $M_{1,2}(t_Z)/M_{1,2}(0)$ due to asymptotic freedom, and these coefficients stand still whatever happens in the sfermion and Higgs sectors of the theory.

A by-product of the invariant (49) is that the phases of the gaugino masses are RG invariants (like that of the μ parameter). However, this is correct only at one-loop level; at two loops the phases of the trilinear couplings disturb the relation between IR and UV phases of the gaugino masses.

Another invariant of mass dim-1 is related with the B parameter for which we obtain:

$$\begin{aligned} I_3 &= B - \frac{27}{61}A_t - \frac{21}{61}A_b - \frac{10}{61}A_\tau - \frac{256}{183}M_3 - \frac{9}{61}M_2 + \frac{73}{2013}M_1 \\ &+ c_1 A'_t + c_2 A'_b + c_3 A'_\tau - (c_1 + c_2 + c_3)\mu', \end{aligned} \tag{51}$$

with arbitrary coefficients c_i such that in the limit $A'_{t,b,\tau}, \mu' \to \mu$ it reproduces the well known MSSM invariant which can be expressed in terms of other parameters

$$\begin{aligned} B(t) &= B(0) + \frac{27}{61}\left(A_t(t) - A_t(0)\right) + \frac{21}{61}\left(A_b(t) - A_b(0)\right) + \frac{10}{61}\left(A_\tau(t) - A_\tau(0)\right) \\ &+ \frac{256}{183}M_3(0)\left(\frac{g_3(t)^2}{g_3(0)^2} - 1\right) + \frac{9}{61}M_2(0)\left(\frac{g_2(t)^2}{g_2(0)^2} - 1\right) - \frac{73}{2013}M_1(0)\left(\frac{g_1(t)^2}{g_1(0)^2} - 1\right). \end{aligned} \tag{52}$$

Concerning mass dimension-2 terms we obtain a general invariant relation in the NHSSM by brute force as follows

$$\begin{aligned} I_4 &= \left(\frac{c_1}{6} + \frac{9c_2}{16} + \frac{c_3}{2} + \frac{c_4}{2}\right) m^2_{H_u}(t) + \left(\frac{-c_1}{6} + \frac{3c_2}{16} - \frac{c_3}{2} - \frac{c_4}{2}\right) m^2_{H_d}(t) \\ &+ \left(\frac{c_1}{2} - \frac{9c_2}{16} - \frac{c_3}{2} + \frac{3c_4}{2}\right) m^2_{t_L}(t) + \left(\frac{-c_1}{2} - \frac{9c_2}{16} - \frac{c_3}{2} - \frac{3c_4}{2}\right) m^2_{t_R}(t) \\ &+ \left(\frac{c_1}{6} - \frac{3c_2}{16} + \frac{c_3}{2} - \frac{3c_4}{2}\right) m^2_{l_L}(t) + c_3 m^2_{b_R}(t) + c_4 m^2_{l_R}(t) \\ &- \left(\frac{c_1}{33} + \frac{c_2}{44}\right) M_1^2(t) + c_1 M_2^2(t) + c_2 M_3^2(t) \\ &+ c_5 A'^2_t(t) + c_6 A'^2_b(t) + c_7 A'^2_\tau(t) - \left(\frac{3c_2}{4} + c_5 + c_6 + c_7\right)\mu'^2(t). \end{aligned} \tag{53}$$

where c_i are arbitrary constants. To visualize our results lets set all coefficient to zero but $c_{5,6,7}$ we then obtain

$$c_5 A'^2_t(t) + c_6 A'^2_b(t) + c_7 A'^2_\tau(t) - (c_5 + c_6 + c_7)\mu'^2(t), \tag{54}$$

which is obviously invariant in the limit $A'_{t,b,\tau}, \mu' \to \mu$. Note that using this limiting case one can obtain another invariant, when supplemented with $m^2_{H_{u,d}}(t) \to m^2_{H_{u,d}}(t) + \mu^2(t)$ brings the most general form of MSSM invariant mass of dim-2. In the cases when we relax these substitutions we obtain more general structures. Now we vary the coefficients of various soft masses for constructing invariants in terms of M_i and μ parameters. Using this freedom, when we set $c_1 = -3$, $c_4 = 1$ and all other coefficients to zero, we get

$$I_5 = -2m_{lL}^2(t) + m_{lR}^2(t) + 3\,|M_2(t)|^2 - \frac{1}{11}\,|M_1(t)|^2 \tag{55}$$

and similarly various patterns of the coefficients give rise to

$$\begin{aligned}
I_6 &= m_{H_u}^2(t) - \frac{3}{2}m_{t_R}^2(t) + \frac{4}{3}\,|M_3(t)|^2 + \frac{3}{2}\,|M_2(t)|^2 - \frac{5}{66}\,|M_1(t)|^2 - |\mu'(t)|^2\,, \\
I_7 &= m_{H_d}^2(t) - \frac{3}{2}m_{b_R}^2(t) - m_{l_L}^2(t) + \frac{4}{3}\,|M_3(t)|^2 - \frac{1}{33}\,|M_1(t)|^2 - |\mu'(t)|^2\,, \\
I_8 &= m_{t_R}^2(t) + m_{b_R}^2(t) - 2m_{t_L}^2(t) - 3\,|M_2(t)|^2 + \frac{1}{11}\,|M_1(t)|^2\,, \\
I_9 &= m_{H_u}^2(t) + m_{H_d}^2(t) - 3m_{t_L}^2(t) - m_{l_L}^2(t) \\
&+ \frac{8}{3}\,|M_3(t)|^2 - 3\,|M_2(t)|^2 + \frac{1}{33}\,|M_1(t)|^2 - 2\,|\mu'(t)|^2\,, \\
I_{10} &= m_{H_d}^2(t) - \frac{3}{2}m_{b_R}^2(t) - \frac{3}{2}m_{l_L}^2(t) + \frac{1}{4}m_{l_R}^2(t) \\
&+ \frac{4}{3}\,|M_3(t)|^2 - \frac{3}{4}\,|M_2(t)|^2 + \frac{1}{132}\,|M_1(t)|^2 - |\mu'(t)|^2\,,
\end{aligned} \tag{56}$$

which should be compared with the results of (see [32]). Clearly, one can construct new invariants by combining the ones presented here or by varying the coefficients expressed as c_i. Although the results presented here and the results of [32] coincide a term is observed to be missing in some of the invariant equations. This stems from the definitions and frameworks *i.e.* we work within minimal supergravity (with non-holomorphic soft terms). Here we confirm the results of [31, 32] in certain limits and we also generate new invariants.

The general form (53) and the invariants that follow could be very useful for sparticle spectroscopy [16] in that they provide scale-invariant correlations among various sparticle masses. All the invariants presented here show non-anomalous behaviors unless they bear μ' terms. As an example lets take I_9 Fig. (8), which demonstrates the fixed behavior. Notice that while it is scale-dependent, it is still very useful since its dependency is very soft. However, notice that they are obtained without noting flavor mixing and in the mSUGRA framework. Nevertheless, using them one can (*i*) test the internal consistency of the model while fitting to the experimental data; (*ii*) rehabilitate poorly known parameters supplementing the well-measured ones; (*iii*) determine what kind of supersymmetry breaking mechanism is realized in Nature; and finally (*iv*) separately examine the UV scale configurations of the trilinear couplings as they do not explicitly contribute to the invariants.

Consequently, if one single invariant is measured then all are done, and in case the experimental data prefer a certain correlation pattern among the invariants then the corresponding UV scale model is preferred. In this sense, rendering unnecessary the RG running of individual sparticle masses up to the messenger scale, the invariants speed up the determination of what kind of supersymmetry breaking mechanism is realized in Nature.

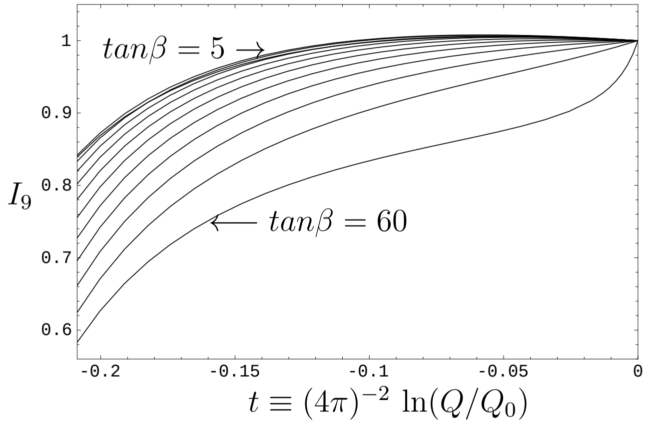

Figure 8: Fixed point behavior of the anomal I_9 against scale. Here we assume same weight for all soft terms ($\sim$ 40 GeV) and re-scale the figure (initial value of this invariant is $\sim 32\,TeV^2$).

5 Conclusion

It is important to explore the features of MSSM and its extensions as general as possible. This will be clear as experimental data accumulates about the masses of all predicted particles, and for the time being it should be calculated at low energies using the RGEs. For that aim NHSSM offers novel opportunities which should be studied in more detail. Compared with its enrichments, there are not enough papers in the literature about the phenomenological consequences of the NHSSM. So we try to cover this issue from many sides. Because we do not know the mechanism of supersymmetry breaking, extensions of the MSSM should be taken seriously to ease the shortcomings of the MSSM. In this paper we explored the main features of NHSSM with minimal particle content and observe that, in addition to mimic the reactions of the MSSM (like gauginos or Yukawa couplings), NHSSM offers interesting opportunities. Even, under certain assumptions, it is possible to completely get rid of famous μ problem in the NHSSM, and this corresponds to two special turning points in low and high $tan\beta$ regimes, which is not possible in classical MSSM. The price that must be paid is, facing additional primed trilinear coupling and fine tuning of parameters for GUT boundaries.

One of the main results of this work is to present semi-analytic solutions of RGEs of NHSSM which enables one to study the phenomenology in detail. Using the solutions presented here one can investigate the reaction of the NHSSM deeper. Notice that the solutions presented in the Appendices have nonzero phases which should be used to go deeper in the phenomenology.

Another result is to present a general form of RGIs which can be used to derive new relations in addition to those existing in the literature. We observed that by using existing RGEs one can construct RGIs with a simple computer code which indeed offers a very practical way of handling the equations. These invariants turn out to be highly useful in making otherwise indirect relations among the parameters manifest. Moreover, they serve as efficient tools for performing fast consistency checks for deriving poorly known parameters from known ones in course of fitting the model to experimental data, and for probing the mechanism that breaks the supersymmetry.

6 Acknowledgement

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A Explicit form of RGEs of the NHSSM

For the NHSSM one-loop renormalization group equations can be found in [10] we also present here for the sake of completeness.

$$\begin{aligned}\beta_{m^2_{H_d}} &= 2h_\tau^2(m^2_{H_d}+A_\tau^2+m^2_{l_L}+m^2_{l_R})+6h_b^2(m^2_{H_d}+A_b^2+m^2_{t_L}+m^2_{b_R})\\ &+ 6h_t^2\ A^2_{t'}-8C_H\mu'^2-6g_2^2M_2^2-2g'^2M_1^2,\end{aligned} \tag{57}$$

$$\begin{aligned}\beta_{m^2_{H_u}} &= 6h_t^2(m^2_{H_u}+A_t^2+m^2_{t_L}+m^2_{t_R})+2h_\tau^2\ A^2_{\tau'}+6h_b^2A^2_{b'}\\ &- 8C_H\mu'^2-6g_2^2\ M_2^2-2g'^2M_1^2,\end{aligned} \tag{58}$$

$$\begin{aligned}\beta_{m^2_3} &= (h_\tau^2+3h_b^2+3h_t^2)m_3^2+2h_\tau^2\ A_{\tau'}A_\tau+6h_b^2A_{b'}A_b+6h_t^2\ A_{t'}A_t\\ &- 4C_Hm_3^2+6g_2^2\mu' M_2+2\ g'^2M_1\mu',\end{aligned} \tag{59}$$

$$\beta_{\mu'} = (h_\tau^2+3h_b^2+3h_t^2-4C_H)\mu', \tag{60}$$

$$\beta_{A_{\tau'}} = (h_\tau^2-3h_b^2+3h_t^2)A_{\tau'}+6h_b^2A_{b'}+(4A_{\tau'}-8\mu')C_H, \tag{61}$$

$$\beta_{A_\tau} = 8h_\tau^2A_\tau+6h_b^2A_b+6g_2^2M_2+6\ g'^2M_1, \tag{62}$$

$$\beta_{A_{b'}} = (-h_\tau^2+3h_b^2+h_t^2)A_{b'}+2A_{\tau'}h_\tau^2-2h_t^2(A_{t'}-2\mu')+(4A_{b'}-8\mu')C_H, \tag{63}$$

$$\beta_{A_b} = 2h_\tau^2A_\tau+12h_b^2A_b+2h_t^2A_t+\frac{32}{3}g_3^2M_3+6g_2^2M_2+\frac{14}{9}g'^2M_1, \tag{64}$$

$$\beta_{A_{t'}} = (h_\tau^2+h_b^2+3h_t^2)A_{t'}-2A_{b'}h_b^2+4\mu' h_b^2+(4A_{t'}-8\mu')C_H, \tag{65}$$

$$\beta_{A_t} = 2h_b^2A_b+12h_t^2A_t+\frac{32}{3}g_3^2M_3+6\ g_2^2M_2+\frac{26}{9}g'^2M_1, \tag{66}$$

$$\begin{aligned}\beta_{m^2_{t_L}} &= 2h_b^2(m^2_{t_L}+m^2_{b_R}+m^2_{H_d}+A^2_{b'}+A_b^2-2\mu'^2)\\ &+ 2h_t^2\ (m^2_{t_L}+m^2_{t_R}+m^2_{H_u}+A^2_{t'}+A_t^2-2\mu'^2)\\ &- \frac{32}{3}g_3^2M_3^2-6g_2^2M_2^2-\frac{2}{9}g'^2M_1^2,\end{aligned} \tag{67}$$

$$\beta_{m^2_{t_R}} = 4h_t^2(m^2_{t_L}+m^2_{t_R}+m^2_{H_u}+A^2_{t'}+A_t^2-2\mu'^2)-\frac{32}{3}g_3^2M_3^2-\frac{32}{9}\ g'^2M_1^2, \tag{68}$$

$$\beta_{m^2_{b_R}} = 4h_b^2(m^2_{t_L}+m^2_{b_R}+m^2_{H_d}+A^2_{b'}+A_b^2-2\mu'^2)-\frac{32}{3}g_3^2M_3^2-\frac{8}{9}g'^2\ M_1^2, \tag{69}$$

$$\beta_{m^2_{l_L}} = 2h_\tau^2(m^2_{l_L} + m^2_{l_R} + m^2_{H_d} + A^2_{\tau'} + A^2_\tau - 2\mu'^2) - 6g_2^2M_2^2 - 2g'^2M_1^2, \quad (70)$$

$$\beta_{m^2_{l_R}} = 4h_\tau^2(m^2_{l_L} + m^2_{l_R} + m^2_{H_d} + A^2_{\tau'} + A^2_\tau - 2\mu'^2) - 8g'^2M_1^2, \quad (71)$$

$$\beta_{M_i} = 2b_iM_ig_i^2, \quad (72)$$

here $b_{1,2,3} = (\frac{33}{5}, 1, -3)$, $g'^2 = \frac{3}{5}g_1^2$, $C_H = \frac{3}{4}g_2^2 + \frac{3}{20}g_1^2$, $M_{GUT} = 1.4 \times 10^{16}$ GeV and $M_Z \leq Q \leq M_{GUT}$. By assuming that the SUSY is broken with non-standard soft terms; we obtained semi-analytic solutions for all soft terms through the one-loop RGEs given above and express our results at the electro-weak scale in terms of GUT scale parameters. Our results are presented for moderate ($tan\beta$=5) and large ($tan\beta$=50) choices.

B Solutions of mass squared & trilinear terms in the NHSSM

Using low ($tan\beta = 5$) and high ($tan\beta = 50$) values of $tan\beta$, the most general form of the mass-squared and trilinear terms can be written in terms of boundary conditions of gauge coupling unification scale which is roughly $M_{GUT} \sim 10^{17}$ GeV. Notice that our phase convention is to assign $1, 2, 3$ and 4 for M_1, M_2, M_3 and μ'; for other quantities it is obvious and can be inferred from the multipliers.

B.1 Low $tan\beta$ regime

$$\begin{aligned}
m^2_{H_u}(t_Z) &= 0.000216A^2_{b_0} - 1.59 \times 10^{-7}A_{b_0}A_{\tau_0}cos\phi_{b\tau} - 0.0000203A_{b_0}M_{10}cos\phi_{b1} \\
&- 0.000191A_{b_0}M_{20}cos\phi_{b2} - 0.000857A_{b_0}M_{30}cos\phi_{b3} - 0.00124A^2_{b'_0} \\
&+ 1.73 \times 10^{-6}A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'} - 0.000563A_{b'_0}\mu'_0cos\phi_{b'4} - 0.0869A^2_{t_0} \\
&+ 0.0000648A_{t_0}A_{b_0}cos\phi_{tb} - 2.05 \times 10^{-8}A_{t_0}A_{\tau_0}cos\phi_{t\tau} + 0.0109A_{t_0}M_{10}cos\phi_{t1} \\
&+ 0.0672A_{t_0}M_{20}cos\phi_{t2} + 0.302A_{t_0}M_{30}cos\phi_{t3} - 7.96 \times 10^{-8}A^2_{\tau_0} \\
&+ 2.25 \times 10^{-8}A_{\tau_0}M_{10}cos\phi_{\tau1} + 1.19 \times 10^{-7}A_{\tau_0}M_{20}cos\phi_{\tau2} + 4.14 \times 10^{-7}A_{\tau_0}M_{30}cos\phi_{\tau3} \\
&- 0.000287A^2_{\tau'_0} - 0.000248A_{\tau'_0}\mu'_0cos\phi_{\tau',4} + 0.105A^2_{t'_0} \\
&- 0.000284A_{t'_0}A_{b'_0}cos\phi_{t'b'} + 2.25 \times 10^{-7}A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'} + 0.0674A_{t'_0}\mu'_0cos\phi_{t'4} \\
&+ 0.00106M^2_{10} - 0.0058M_{10}M_{20}cos\phi_{12} - 0.0291M_{10}M_{30}cos\phi_{13} \\
&+ 0.187M^2_{20} - 0.206M_{20}M_{30}cos\phi_{23} - 2.79M^2_{30} \\
&+ 0.000217m^2_{b_R0} + 0.000217m^2_{H_d0} + 0.612m^2_{H_u0} \\
&- 7.98 \times 10^{-8}m^2_{l_L0} - 8. \times 10^{-8}m^2_{l_R0} - 0.388m^2_{t_L0} \\
&- 0.388m^2_{t_R0} + 0.136\mu'^2_0, \qquad (73)
\end{aligned}$$

$$m^2_{H_d}(t_Z) = -0.0032A^2_{b_0} + 5 \times 10^{-6}A_{b_0}A_{\tau_0}cos\phi_{b\tau} + 0.00018A_{b_0}M_{10}cos\phi_{b1}$$

$$
\begin{aligned}
&+ \ 0.0022A_{b_0}M_{20}cos\phi_{b2} + 0.01A_{b_0}M_{30}cos\phi_{b3} + 2.9\times10^{-6}A^2_{b'_0}\\
&- \ 2.4\times10^{-9}A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'} - 0.00017A_{b'_0}\mu'_0cos\phi_{b'4} + 0.00008A^2_{t_0}\\
&+ \ 0.00058A_{t_0}A_{b_0}cos\phi_{tb} - 4.7\times10^{-7}A_{t_0}A_{\tau_0}cos\phi_{t\tau} - 0.000028A_{t_0}M_{10}cos\phi_{t1}\\
&- \ 0.00029A_{t_0}M_{20}cos\phi_{t2} - 0.0013A_{t_0}M_{30}cos\phi_{t3} - 0.00078A^2_{\tau_0}\\
&+ \ 0.00018A_{\tau_0}M_{10}cos\phi_{\tau1} + 0.0005A_{\tau_0}M_{20}cos\phi_{\tau2} - 7.9\times10^{-6}A_{\tau_0}M_{30}cos\phi_{\tau3}\\
&+ \ 5.2\times10^{-7}A^2_{\tau'_0} + 3.5\times10^{-7}A_{\tau'_0}\mu'_0cos\phi_{\tau'4} - 0.37A^2_{t'_0}\\
&- \ 0.00026A_{t'_0}A_{b'_0}cos\phi_{t'b'} + 7.5\times10^{-8}A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'} - 0.31A_{t'_0}\mu'_0cos\phi_{t'4}\\
&+ \ 0.037M^2_{10} - 0.00013M_{10}M_{20}cos\phi_{12} - 0.0003M_{10}M_{30}cos\phi_{13}\\
&+ \ 0.48M^2_{20} - 0.004M_{20}M_{30}cos\phi_{23} - 0.026M^2_{30}\\
&- \ 0.0032m^2_{b_R0} + m^2_{H_d0} + 0.00029m^2_{H_u0}\\
&- \ 0.00079m^2_{l_L0} - 0.00079m^2_{l_R0} - 0.0029m^2_{t_L0}\\
&+ \ 0.00029m^2_{t_R0} + 0.6\mu'^2_0, \qquad (74)
\end{aligned}
$$

$$
\begin{aligned}
m^2_{t_L}(t_Z) \ = \ & -0.00099A^2_{b_0} + 9.8\times10^{-7}A_{b_0}A_{\tau_0}cos\phi_{b\tau} + 0.000053A_{b_0}M_{10}cos\phi_{b1}\\
&+ \ 0.00068A_{b_0}M_{20}cos\phi_{b2} + 0.003A_{b_0}M_{30}cos\phi_{b3} - 0.00041A^2_{b'_0}\\
&+ \ 3.1\times10^{-7}A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'} - 0.00024A_{b'_0}\mu'_0cos\phi_{b'4} - 0.029A^2_{t_0}\\
&+ \ 0.00022A_{t_0}A_{b_0}cos\phi_{tb} - 1.2\times10^{-7}A_{t_0}A_{\tau_0}cos\phi_{t\tau} + 0.0036A_{t_0}M_{10}cos\phi_{t1}\\
&+ \ 0.022A_{t_0}M_{20}cos\phi_{t2} + 0.1A_{t_0}M_{30}cos\phi_{t3} + 4.9\times10^{-7}A^2_{\tau_0}\\
&- \ 1.3\times10^{-7}A_{\tau_0}M_{10}cos\phi_{\tau1} - 6.4\times10^{-7}A_{\tau_0}M_{20}cos\phi_{\tau2} - 1.9\times10^{-6}A_{\tau_0}M_{30}cos\phi_{\tau3}\\
&+ \ 0.000039A^2_{\tau'_0} + 0.000024A_{\tau'_0}\mu'_0cos\phi_{\tau'4} - 0.089A^2_{t'_0}\\
&- \ 0.00018A_{t'_0}A_{b'_0}cos\phi_{t'b'} + 7.7\times10^{-8}A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'} - 0.08A_{t'_0}\mu'_0cos\phi_{t'4}\\
&- \ 0.0081M^2_{10} - 0.002M_{10}M_{20}cos\phi_{12} - 0.0098M_{10}M_{30}cos\phi_{13}\\
&+ \ 0.38M^2_{20} - 0.07M_{20}M_{30}cos\phi_{23} + 5.4M^2_{30}\\
&- \ 0.00099m^2_{b_R0} - 0.00099m^2_{H_d0} - 0.13m^2_{H_u0}\\
&+ \ 4.9\times10^{-7}m^2_{l_L0} + 4.9\times10^{-7}m^2_{l_R0} + 0.87m^2_{t_L0}\\
&- \ 0.13m^2_{t_R0} + 0.3\mu'^2_0, \qquad (75)
\end{aligned}
$$

$$
\begin{aligned}
m^2_{t_R}(t_Z) \ = \ & 0.00014A^2_{b_0} - 1.1\times10^{-7}A_{b_0}A_{\tau_0}cos\phi_{b\tau} - 0.000014A_{b_0}M_{10}cos\phi_{b1}\\
&- \ 0.00013A_{b_0}M_{20}cos\phi_{b2} - 0.00057A_{b_0}M_{30}cos\phi_{b3} + 0.00037A^2_{b'_0}\\
&- \ 3.2\times10^{-7}A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'} + 0.000042A_{b'_0}\mu'_0cos\phi_{b'4} - 0.058A^2_{t_0}\\
&+ \ 0.000043A_{t_0}A_{b_0}cos\phi_{tb} - 1.4\times10^{-8}A_{t_0}A_{\tau_0}cos\phi_{t\tau} + 0.0072A_{t_0}M_{10}cos\phi_{t1}\\
&+ \ 0.045A_{t_0}M_{20}cos\phi_{t2} + 0.2A_{t_0}M_{30}cos\phi_{t3} - 5.3\times10^{-8}A^2_{\tau_0}\\
&+ \ 1.5\times10^{-8}A_{\tau_0}M_{10}cos\phi_{\tau1} + 8.\times10^{-8}A_{\tau_0}M_{20}cos\phi_{\tau2} + 2.8\times10^{-6}A_{\tau_0}M_{30}cos\phi_{\tau3}\\
&+ \ 0.000077A^2_{\tau'_0} + 0.000048A_{\tau'_0}\mu'_0cos\phi_{\tau'4} - 0.18A^2_{t'_0}\\
&- \ 0.000073A_{t'_0}A_{b'_0}cos\phi_{t'b'} + 7.7\times10^{-9}A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'} - 0.16A_{t'_0}\mu'_0cos\phi_{t'4}
\end{aligned}
$$

$$
\begin{aligned}
&+ \ 0.043M_{10}^2 - 0.0039M_{10}M_{20}cos\phi_{12} - 0.019M_{10}M_{30}cos\phi_{13} \\
&- \ 0.2M_{20}^2 - 0.14M_{20}M_{30}cos\phi_{23} + 4.4M_{30}^2 \\
&+ \ 0.00014m_{b_R0}^2 + 0.00014m_{H_d0}^2 - 0.26m_{H_u0}^2 \\
&- \ 5.3\times10^{-8}m_{l_L0}^2 - 5.3\times10^{-8}m_{l_R0}^2 - 0.26m_{t_L0}^2 \\
&+ \ 0.74m_{t_R0}^2 + 0.6\mu_0'^2, \qquad (76)
\end{aligned}
$$

$$
\begin{aligned}
m_{b_R}^2(t_Z) \ = \ & -0.0021A_{b_0}^2 + 2.1\times10^{-6}A_{b_0}A_{\tau_0}cos\phi_{b\tau} + 0.00012A_{b_0}M_{10}cos\phi_{b1} \\
&+ \ 0.0015A_{b_0}M_{20}cos\phi_{b2} + 0.0066A_{b_0}M_{30}cos\phi_{b3} - 0.0012A_{b_0'}^2 \\
&+ \ 9.5\times10^{-7}A_{b_0'}A_{\tau_0'}cos\phi_{b'\tau'} - 0.00053A_{b_0'}\mu_0'cos\phi_{b'4} + 0.000053A_{t_0}^2 \\
&+ \ 0.00039A_{t_0}A_{b_0}cos\phi_{tb} - 2.2\times10^{-7}A_{t_0}A_{\tau_0}cos\phi_{t\tau} - 0.000019A_{t_0}M_{10}cos\phi_{t1} \\
&- \ 0.00019A_{t_0}M_{20}cos\phi_{t2} - 0.00086A_{t_0}M_{30}cos\phi_{t3} + 1\times10^{-6}A_{\tau_0}^2 \\
&- \ 2.7\times10^{-7}A_{\tau_0}M_{10}cos\phi_{\tau1} - 1.4\times10^{-6}A_{\tau_0}M_{20}cos\phi_{\tau2} \ - 4.1\times10^{-6}A_{\tau_0}M_{30}cos\phi_{\tau3} \\
&- \ 4.5\times10^{-8}A_{\tau_0'}^2 + 2.3\times10^{-7}A_{\tau_0'}\mu_0'cos\phi_{\tau'4} + 0.00068A_{t_0'}^2 \\
&- \ 0.00029A_{t_0'}A_{b_0'}cos\phi_{t'b'} + 1.5\times10^{-7}A_{t_0'}A_{\tau_0'}cos\phi_{t'\tau'} + 0.00038A_{t_0'}\mu_0'cos\phi_{t'4} \\
&+ \ 0.017M_{10}^2 - 0.000042M_{10}M_{20}cos\phi_{12} - 0.0002M_{10}M_{30}cos\phi_{13} \\
&- \ 0.0017M_{20}^2 - 0.0027M_{20}M_{30}cos\phi_{23} + 6.3M_{30}^2 \\
&+ \ 1m_{b_R0}^2 - 0.0021m_{H_d0}^2 + 0.0002m_{H_u0}^2 \\
&+ \ 1\times10^{-6}m_{l_L0}^2 + 1\times10^{-6}m_{l_R0}^2 - 0.0019m_{t_L0}^2 \\
&+ \ 0.0002m_{t_R0}^2 + 0.0029\mu_0'^2, \qquad (77)
\end{aligned}
$$

$$
\begin{aligned}
m_{l_L}^2(t_Z) \ = \ & 9.6\times10^{-7}A_{b_0}^2 + 1.9\times10^{-6}A_{b_0}A_{\tau_0}cos\phi_{b\tau} - 3.1\times10^{-7}A_{b_0}M_{10}cos\phi_{b1} \\
&- \ 1.3\times10^{-6}A_{b_0}M_{20}cos\phi_{b2} - 1.8\times10^{-6}A_{b_0}M_{30}cos\phi_{b3} - 1.3\times10^{-9}A_{b_0'}^2 \\
&+ \ 7.9\times10^{-7}A_{b_0'}A_{\tau_0'}cos\phi_{b'\tau'} + 5.4\times10^{-7}A_{b_0'}\mu_0'cos\phi_{b'4} - 2.6\times10^{-8}A_{t_0}^2 \\
&- \ 1.3\times10^{-7}A_{t_0}A_{b_0}cos\phi_{tb} - 1.3\times10^{-7}A_{t_0}A_{\tau_0}cos\phi_{t\tau} + 2.5\times10^{-8}A_{t_0}M_{10}cos\phi_{t1} \\
&+ \ 1.1\times10^{-7}A_{t_0}M_{20}cos\phi_{t2} + 2.1\times10^{-7}A_{t_0}M_{30}cos\phi_{t3} - 0.00079A_{\tau_0}^2 \\
&+ \ 0.00018A_{\tau_0}M_{10}cos\phi_{\tau1} + 0.0005A_{\tau_0}M_{20}cos\phi_{\tau2} - 1.8\times10^{-6}A_{\tau_0}M_{30}cos\phi_{\tau3} \\
&- \ 0.0004A_{\tau_0'}^2 - 0.00032A_{\tau_0'}\mu_0'cos\phi_{\tau'4} + 0.00018A_{t_0'}^2 \\
&+ \ 6.9\times10^{-8}A_{t_0'}A_{b_0'}cos\phi_{t'b'} + 6.8\times10^{-8}A_{t_0'}A_{\tau_0'}cos\phi_{t'\tau'} + 0.0001A_{t_0'}\mu_0'cos\phi_{t'4} \\
&+ \ 0.038M_{10}^2 - 0.000066M_{10}M_{20}cos\phi_{12} + 3.2\times10^{-7}M_{10}M_{30}cos\phi_{13} \\
&+ \ 0.48M_{20}^2 + 1.5\times10^{-6}M_{20}M_{30}cos\phi_{23} + 4.\times10^{-6}M_{30}^2 \\
&+ \ 9.7\times10^{-7}m_{b_R0}^2 - 0.00079m_{H_d0}^2 - 6.6\times10^{-8}m_{H_u0}^2 \\
&+ \ 1m_{l_L0}^2 - 0.00079m_{l_R0}^2 + 9.\times10^{-7}m_{t_L0}^2 \\
&- \ 6.6\times10^{-8}m_{t_R0}^2 + 0.0012\mu_0'^2, \qquad (78)
\end{aligned}
$$

$$
m_{l_R}^2(t_Z) \ = \ 1.9\times10^{-6}A_{b_0}^2 + 3.9\times10^{-6}A_{b_0}A_{\tau_0}cos\phi_{b\tau} - 6.3\times10^{-7}A_{b_0}M_{10}cos\phi_{b1}
$$

$$
\begin{aligned}
&- && 2.6\times10^{-6}A_{b_0}M_{20}cos\phi_{b2}-3.7\times10^{-6}A_{b_0}M_{30}cos\phi_{b3}-2.6\times10^{-9}A_{b'_0}^2\\
&+ && 1.6\times10^{-6}A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'}+1.1\times10^{-6}A_{b'_0}\mu'_0cos\phi_{b'4}-5.3\times10^{-8}A_{t_0}^2\\
&- && 2.6\times10^{-7}A_{t_0}A_{b_0}cos\phi_{tb}-2.7\times10^{-7}A_{t_0}A_{\tau_0}cos\phi_{t\tau}+5.1\times10^{-8}A_{t_0}M_{10}cos\phi_{t1}\\
&+ && 2.3\times10^{-7}A_{t_0}M_{20}cos\phi_{t2}+4.1\times10^{-7}A_{t_0}M_{30}cos\phi_{t3}-0.0016A_{\tau_0}^2\\
&+ && 0.00035A_{\tau_0}M_{10}cos\phi_{\tau1}+0.001A_{\tau_0}M_{20}cos\phi_{\tau2}-3.7\times10^{-6}A_{\tau_0}M_{30}cos\phi_{\tau3}\\
&- && 0.00081A_{\tau'_0}^2-0.00064A_{\tau'_0}\mu'_0cos\phi_{\tau'4}+0.00035A_{t'_0}^2\\
&+ && 1.4\times10^{-7}A_{t'_0}A_{b'_0}cos\phi_{t'b'}+1.4\times10^{-7}A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'}+0.0002A_{t'_0}\mu'_0cos\phi_{t'4}\\
&+ && 0.15M_{10}^2-0.00013M_{10}M_{20}cos\phi_{12}+6.4\times10^{-7}M_{10}M_{30}cos\phi_{13}\\
&- && 0.0011M_{20}^2+2.9\times10^{-6}M_{20}M_{30}cos\phi_{23}-0.00019M_{30}^2\\
&+ && 1.9\times10^{-6}m_{b_R0}^2-0.0016m_{H_d0}^2-1.3\times10^{-7}m_{H_u0}^2\\
&- && 0.0016m_{l_L0}^2+m_{l_R0}^2+1.8\times10^{-6}m_{t_L0}^2\\
&- && 1.3\times10^{-7}m_{t_R0}^2+0.0025\mu_0'^2 \qquad (79)
\end{aligned}
$$

$$
\begin{aligned}
m_3^2(t_Z) &= && 0.00012A_{b'_0}A_{t_0}cos\phi_{b't}+1.7\times10^{-6}A_{b'_0}A_{\tau_0}cos\phi_{b'\tau}+0.000069A_{b'_0}M_{10}cos\phi_{b'1}\\
&+ && 0.0008A_{b'_0}M_{20}cos\phi_{b'2}+0.0036A_{b'_0}M_{30}cos\phi_{b'3}+1.5\times10^{-6}A_{\tau'_0}A_{b_0}cos\phi_{\tau'b}\\
&- && 1.2\times10^{-7}A_{\tau'_0}A_{t_0}cos\phi_{\tau't}-0.00052A_{\tau'_0}A_{\tau_0}cos\phi_{\tau'\tau}+0.000054A_{\tau'_0}M_{10}cos\phi_{\tau'1}\\
&+ && 0.00015A_{\tau'_0}M_{20}cos\phi_{\tau'2}-2.4\times10^{-6}A_{\tau'_0}M_{30}cos\phi_{\tau'3}-0.00017A_{t'_0}A_{b_0}cos\phi_{t'b}\\
&- && 0.27A_{t'_0}A_{t_0}cos\phi_{t't}+1.9\times10^{-7}A_{t'_0}A_{\tau_0}cos\phi_{t'\tau}+0.015A_{t'_0}M_{10}cos\phi_{t'1}\\
&+ && 0.092A_{t'_0}M_{20}cos\phi_{t'2}+0.39A_{t'_0}M_{30}cos\phi_{t'3}-0.051M_{10}^2\\
&- && 0.51M_{20}^2+0.96m_{30}^2-0.00044\mu'_0A_{b_0}cos\phi_{4b}\\
&- && 0.098\mu'_0A_{t_0}cos\phi_{4t}-0.00024\mu'_0A_{\tau_0}cos\phi_{4\tau}+0.0079\mu'_0M_{10}cos\phi_{41}\\
&+ && 0.052\mu'_0M_{20}cos\phi_{42}+0.26\mu'_0M_{30}cos\phi_{43}, \qquad (80)
\end{aligned}
$$

B.2 High $tan\beta$ regime

$$
\begin{aligned}
m_{H_u}^2(t_Z) &= && 0.014A_{b_0}^2-0.0012A_{b_0}A_{\tau_0}cos\phi_{b\tau}-0.0017A_{b_0}M_{10}cos\phi_{b1}\\
&- && 0.014A_{b_0}M_{20}cos\phi_{b2}-0.065A_{b_0}M_{30}cos\phi_{b3}-0.18A_{b'_0}^2\\
&+ && 0.044A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'}-0.035A_{b'_0}\mu'_0cos\phi_{b'4}-0.083A_{t_0}^2\\
&+ && 0.01A_{t_0}A_{b_0}cos\phi_{tb}-0.00053A_{t_0}A_{\tau_0}cos\phi_{t\tau}+0.01A_{t_0}M_{10}cos\phi_{t1}\\
&+ && 0.06A_{t_0}M_{20}cos\phi_{t2}+0.27A_{t_0}M_{30}cos\phi_{t3}-0.0011A_{\tau_0}^2\\
&+ && 0.00028A_{\tau_0}M_{10}cos\phi_{\tau1}+0.0014A_{\tau_0}M_{20}cos\phi_{\tau2}+0.0049A_{\tau_0}M_{30}cos\phi_{\tau3}\\
&- && 0.056A_{\tau'_0}^2-0.031A_{\tau'_0}\mu'_0cos\phi_{\tau',4}+0.096A_{t'_0}^2\\
&- && 0.032A_{t'_0}A_{b'_0}cos\phi_{t'b'}+0.0049A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'}+0.03A_{t'_0}\mu'_0cos\phi_{t'4}\\
&+ && 0.0013M_{10}^2-0.005M_{10}M_{20}cos\phi_{12}-0.025M_{10}M_{30}cos\phi_{13}\\
&+ && 0.2M_{20}^2-0.17M_{20}M_{30}cos\phi_{23}-2.6M_{30}^2
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 0.029m^2_{b_R0} + 0.028m^2_{H_d0} + 0.6m^2_{H_u0} \\
&- \quad 0.0016m^2_{l_L0} - 0.0016m^2_{l_R0} - 0.37m^2_{t_L0} \\
&- \quad 0.4m^2_{t_R0} - 0.0083\mu'^2_0,
\end{aligned} \tag{81}
$$

$$
\begin{aligned}
m^2_{H_d}(t_Z) &= -0.11A^2_{b_0} + 0.033A_{b_0}A_{\tau_0}cos\phi_{b\tau} + 0.0025A_{b_0}M_{10}cos\phi_{b1} \\
&+ \quad 0.069A_{b_0}M_{20}cos\phi_{b2} + 0.36A_{b_0}M_{30}cos\phi_{b3} + 0.051A^2_{b'_0} \\
&- \quad 0.0074A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'} - 0.00043A_{b'_0}\mu'_0cos\phi_{b'4} + 0.009A^2_{t_0} \\
&+ \quad 0.021A_{t_0}A_{b_0}cos\phi_{tb} - 0.0032A_{t_0}A_{\tau_0}cos\phi_{t\tau} - 0.0013A_{t_0}M_{10}cos\phi_{t1} \\
&- \quad 0.014A_{t_0}M_{20}cos\phi_{t2} - 0.069A_{t_0}M_{30}cos\phi_{t3} - 0.046A^2_{\tau_0} \\
&+ \quad 0.0096A_{\tau_0}M_{10}cos\phi_{\tau1} + 0.018A_{\tau_0}M_{20}cos\phi_{\tau2} - 0.053A_{\tau_0}M_{30}cos\phi_{\tau3} \\
&+ \quad 0.011A^2_{\tau'_0} + 0.0045A_{\tau'_0}\mu'_0cos\phi_{\tau'4} - 0.24A^2_{t'_0} \\
&- \quad 0.025A_{t'_0}A_{b'_0}cos\phi_{t'b'} + 0.001A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'} - 0.11A_{t'_0}\mu'_0cos\phi_{t'4} \\
&+ \quad 0.011M^2_{10} - 0.005M_{10}M_{20}cos\phi_{12} - 0.0055M_{10}M_{30}cos\phi_{13} \\
&+ \quad 0.22M^2_{20} - 0.16M_{20}M_{30}cos\phi_{23} - 2.1M^2_{30} \\
&- \quad 0.31m^2_{b_R0} + 0.61m^2_{H_d0} + 0.03m^2_{H_u0} \\
&- \quad 0.077m^2_{l_L0} - 0.077m^2_{l_R0} - 0.28m^2_{t_L0} \\
&+ \quad 0.03m^2_{t_R0} + 0.13\mu'^2_0,
\end{aligned} \tag{82}
$$

$$
\begin{aligned}
m^2_{t_L}(t_Z) &= -0.036A^2_{b_0} + 0.004A_{b_0}A_{\tau_0}cos\phi_{b\tau} + 0.0013A_{b_0}M_{10}cos\phi_{b1} \\
&+ \quad 0.022A_{b_0}M_{20}cos\phi_{b2} + 0.1A_{b_0}M_{30}cos\phi_{b3} - 0.041A^2_{b'_0} \\
&+ \quad 0.0048A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'} - 0.015A_{b'_0}\mu'_0cos\phi_{b'4} - 0.024A^2_{t_0} \\
&+ \quad 0.011A_{t_0}A_{b_0}cos\phi_{tb} - 0.00083A_{t_0}A_{\tau_0}cos\phi_{t\tau} + 0.0028A_{t_0}M_{10}cos\phi_{t1} \\
&+ \quad 0.015A_{t_0}M_{20}cos\phi_{t2} + 0.067A_{t_0}M_{30}cos\phi_{t3} + 0.0046A^2_{\tau_0} \\
&- \quad 0.00095A_{\tau_0}M_{10}cos\phi_{\tau1} - 0.0042A_{\tau_0}M_{20}cos\phi_{\tau2} - 0.011A_{\tau_0}M_{30}cos\phi_{\tau3} \\
&+ \quad 0.0065A^2_{\tau'_0} + 0.0046A_{\tau'_0}\mu'_0cos\phi_{\tau'4} - 0.052A^2_{t'_0} \\
&- \quad 0.019A_{t'_0}A_{b'_0}cos\phi_{t'b'} + 0.0014A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'} - 0.029A_{t'_0}\mu'_0cos\phi_{t'4} \\
&- \quad 0.011M^2_{10} - 0.0019M_{10}M_{20}cos\phi_{12} - 0.011M_{10}M_{30}cos\phi_{13} \\
&+ \quad 0.32M^2_{20} - 0.11M_{20}M_{30}cos\phi_{23} + 4.7M^2_{30} \\
&- \quad 0.098m^2_{b_R0} - 0.091m^2_{H_d0} - 0.12m^2_{H_u0} \\
&+ \quad 0.0072m^2_{l_L0} + 0.0072m^2_{l_R0} + 0.78m^2_{t_L0} \\
&- \quad 0.12m^2_{t_R0} + 0.35\mu'^2_0,
\end{aligned} \tag{83}
$$

$$
\begin{aligned}
m^2_{t_R}(t_Z) &= 0.0094A^2_{b_0} - 0.00082A_{b_0}A_{\tau_0}cos\phi_{b\tau} - 0.0011A_{b_0}M_{10}cos\phi_{b1} \\
&- \quad 0.0095A_{b_0}M_{20}cos\phi_{b2} - 0.043A_{b_0}M_{30}cos\phi_{b3} + 0.064A^2_{b'_0} \\
&- \quad 0.01A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'} + 0.0051A_{b'_0}\mu'_0cos\phi_{b'4} - 0.055A^2_{t_0}
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 0.0067A_{t_0}A_{b_0}cos\phi_{tb} - 0.00035A_{t_0}A_{\tau_0}cos\phi_{t\tau} + 0.0066A_{t_0}M_{10}cos\phi_{t1} \\
&+ \quad 0.04A_{t_0}M_{20}cos\phi_{t2} + 0.18A_{t_0}M_{30}cos\phi_{t3} - 0.00076A_{\tau_0}^2 \\
&+ \quad 0.00019A_{\tau_0}M_{10}cos\phi_{\tau 1} + 0.00094A_{\tau_0}M_{20}cos\phi_{\tau 2} + 0.0033A_{\tau_0}M_{30}cos\phi_{\tau 3} \\
&+ \quad 0.017A_{\tau_0'}^2 + 0.0073A_{\tau_0'}\mu_0'cos\phi_{\tau' 4} - 0.17A_{t_0'}^2 \\
&- \quad 0.013A_{t_0'}A_{b_0'}cos\phi_{t'b'} + 0.00024A_{t_0'}A_{\tau_0'}cos\phi_{t'\tau'} - 0.078A_{t_0'}\mu_0'cos\phi_{t'4} \\
&+ \quad 0.043M_{10}^2 - 0.0033M_{10}M_{20}cos\phi_{12} - 0.017M_{10}M_{30}cos\phi_{13} \\
&- \quad 0.19M_{20}^2 - 0.11M_{20}M_{30}cos\phi_{23} + 4.6M_{30}^2 \\
&+ \quad 0.02m_{b_R0}^2 + 0.018m_{H_d0}^2 - 0.27m_{H_u0}^2 \\
&- \quad 0.0011m_{l_L0}^2 - 0.0011m_{l_R0}^2 - 0.25m_{t_L0}^2 \\
&+ \quad 0.73m_{t_R0}^2 + 0.43\mu_0'^2
\end{aligned} \tag{84}
$$

$$
\begin{aligned}
m_{b_R}^2(t_Z) \quad = \quad & -0.081A_{b_0}^2 + 0.0089A_{b_0}A_{\tau_0}cos\phi_{b\tau} + 0.0038A_{b_0}M_{10}cos\phi_{b1} \\
+ \quad & 0.053A_{b_0}M_{20}cos\phi_{b2} + 0.25A_{b_0}M_{30}cos\phi_{b3} - 0.15A_{b_0'}^2 \\
+ \quad & 0.02A_{b_0'}A_{\tau_0'}cos\phi_{b'\tau'} - 0.036A_{b_0'}\mu_0'cos\phi_{b'4} + 0.0064A_{t_0}^2 \\
+ \quad & 0.015A_{t_0}A_{b_0}cos\phi_{tb} - 0.0013A_{t_0}A_{\tau_0}cos\phi_{t\tau} - 0.0011A_{t_0}M_{10}cos\phi_{t1} \\
- \quad & 0.01A_{t_0}M_{20}cos\phi_{t2} - 0.047A_{t_0}M_{30}cos\phi_{t3} + 0.01A_{\tau_0}^2 \\
- \quad & 0.0021A_{\tau_0}M_{10}cos\phi_{\tau 1} - 0.0093A_{\tau_0}M_{20}cos\phi_{\tau 2} - 0.025A_{\tau_0}M_{30}cos\phi_{\tau 3} \\
- \quad & 0.0037A_{\tau_0'}^2 + 0.0019A_{\tau_0'}\mu_0'cos\phi_{\tau' 4} + 0.066A_{t_0'}^2 \\
- \quad & 0.025A_{t_0'}A_{b_0'}cos\phi_{t'b'} + 0.0026A_{t_0'}A_{\tau_0'}cos\phi_{t'\tau'} + 0.02A_{t_0'}\mu_0'cos\phi_{t'4} \\
+ \quad & 0.01M_{10}^2 - 0.00056M_{10}M_{20}cos\phi_{12} - 0.0055M_{10}M_{30}cos\phi_{13} \\
- \quad & 0.14M_{20}^2 - 0.11M_{20}M_{30}cos\phi_{23} + 4.9M_{30}^2 \\
+ \quad & 0.78m_{b_R0}^2 - 0.2m_{H_d0}^2 + 0.021m_{H_u0}^2 \\
+ \quad & 0.015m_{l_L0}^2 + 0.015m_{l_R0}^2 - 0.2m_{t_L0}^2 \\
+ \quad & 0.021m_{t_R0}^2 + 0.28\mu_0'^2,
\end{aligned} \tag{85}
$$

$$
\begin{aligned}
m_{l_L}^2(t_Z) \quad = \quad & 0.007A_{b_0}^2 + 0.02A_{b_0}A_{\tau_0}cos\phi_{b\tau} - 0.0032A_{b_0}M_{10}cos\phi_{b1} \\
- \quad & 0.011A_{b_0}M_{20}cos\phi_{b2} - 0.0082A_{b_0}M_{30}cos\phi_{b3} - 0.0049A_{b_0'}^2 \\
+ \quad & 0.023A_{b_0'}A_{\tau_0'}cos\phi_{b'\tau'} + 0.01A_{b_0'}\mu_0'cos\phi_{b'4} - 0.0005A_{t_0}^2 \\
- \quad & 0.00072A_{t_0}A_{b_0}cos\phi_{tb} - 0.0013A_{t_0}A_{\tau_0}cos\phi_{t\tau} + 0.00027A_{t_0}M_{10}cos\phi_{t1} \\
+ \quad & 0.0011A_{t_0}M_{20}cos\phi_{t2} + 0.0015A_{t_0}M_{30}cos\phi_{t3} - 0.062A_{\tau_0}^2 \\
+ \quad & 0.013A_{\tau_0}M_{10}cos\phi_{\tau 1} + 0.032A_{\tau_0}M_{20}cos\phi_{\tau 2} - 0.015A_{\tau_0}M_{30}cos\phi_{\tau 3} \\
- \quad & 0.064A_{\tau_0'}^2 - 0.04A_{\tau_0'}\mu_0'cos\phi_{\tau' 4} + 0.018A_{t_0'}^2 \\
+ \quad & 0.00067A_{t_0'}A_{b_0'}cos\phi_{t'b'} + 0.0016A_{t_0'}A_{\tau_0'}cos\phi_{t'\tau'} + 0.0065A_{t_0'}\mu_0'cos\phi_{t'4} \\
+ \quad & 0.021M_{10}^2 - 0.0042M_{10}M_{20}cos\phi_{12} + 0.0028M_{10}M_{30}cos\phi_{13} \\
+ \quad & 0.43M_{20}^2 + 0.011M_{20}M_{30}cos\phi_{23} + 0.043M_{30}^2
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 0.017m^2_{b_R0} - 0.084m^2_{H_d0} - 0.0011m^2_{H_u0} \\
&+ \quad 0.9m^2_{l_L0} - 0.1m^2_{l_R0} + 0.016m^2_{t_L0} \\
&- \quad 0.0011m^2_{t_R0} + 0.14\mu'^2_0, \qquad (86)
\end{aligned}
$$

$$
\begin{aligned}
m^2_{l_R}(t_Z) \; = \;& 0.014A^2_{b_0} + 0.039A_{b_0}A_{\tau_0}cos\phi_{b\tau} - 0.0063A_{b_0}M_{10}cos\phi_{b1} \\
-& \; 0.023A_{b_0}M_{20}cos\phi_{b2} - 0.016A_{b_0}M_{30}cos\phi_{b3} - 0.0097A^2_{b'_0} \\
+& \; 0.045A_{b'_0}A_{\tau'_0}cos\phi_{b'\tau'} + 0.021A_{b'_0}\mu'_0cos\phi_{b'4} - 0.001A^2_{t_0} \\
-& \; 0.0014A_{t_0}A_{b_0}cos\phi_{tb} - 0.0025A_{t_0}A_{\tau_0}cos\phi_{t\tau} + 0.00053A_{t_0}M_{10}cos\phi_{t1} \\
+& \; 0.0022A_{t_0}M_{20}cos\phi_{t2} + 0.0029A_{t_0}M_{30}cos\phi_{t3} - 0.12A^2_{\tau_0} \\
+& \; 0.025A_{\tau_0}M_{10}cos\phi_{\tau 1} + 0.064A_{\tau_0}M_{20}cos\phi_{\tau 2} - 0.03A_{\tau_0}M_{30}cos\phi_{\tau 3} \\
-& \; 0.13A^2_{\tau'_0} - 0.081A_{\tau'_0}\mu'_0cos\phi_{\tau'4} + 0.035A^2_{t'_0} \\
+& \; 0.0013A_{t'_0}A_{b'_0}cos\phi_{t'b'} + 0.0032A_{t'_0}A_{\tau'_0}cos\phi_{t'\tau'} + 0.013A_{t'_0}\mu'_0cos\phi_{t'4} \\
+& \; 0.12M^2_{10} - 0.0083M_{10}M_{20}cos\phi_{12} + 0.0056M_{10}M_{30}cos\phi_{13} \\
-& \; 0.11M^2_{20} + 0.023M_{20}M_{30}cos\phi_{23} + 0.08M^2_{30} \\
+& \; 0.034m^2_{b_R0} - 0.17m^2_{H_d0} - 0.0022m^2_{H_u0} \\
-& \; 0.2m^2_{l_L0} + 0.8m^2_{l_R0} + 0.031m^2_{t_L0} \\
-& \; 0.0022m^2_{t_R0} + 0.27\mu'^2_0, \qquad (87)
\end{aligned}
$$

$$
\begin{aligned}
m^2_3(t_Z) \; = \;& 0.0052A_{b'_0}A_{t_0}cos\phi_{b't} + 0.024A_{b'_0}A_{\tau_0}cos\phi_{b'\tau} + 0.0035A_{b'_0}M_{10}cos\phi_{b'1} \\
+& \; 0.062A_{b'_0}M_{20}cos\phi_{b'2} + 0.31A_{b'_0}M_{30}cos\phi_{b'3} + 0.022A_{\tau'_0}A_{b_0}cos\phi_{\tau'b} \\
-& \; 0.0016A_{\tau'_0}A_{t_0}cos\phi_{\tau't} - 0.062A_{\tau'_0}A_{\tau_0}cos\phi_{\tau'\tau} + 0.0058A_{\tau'_0}M_{10}cos\phi_{\tau'1} \\
+& \; 0.0098A_{\tau'_0}M_{20}cos\phi_{\tau'2} - 0.037A_{\tau'_0}M_{30}cos\phi_{\tau'3} - 0.0057A_{t'_0}A_{b_0}cos\phi_{t'b} \\
-& \; 0.21A_{t'_0}A_{t_0}cos\phi_{t't} + 0.0019A_{t'_0}A_{\tau_0}cos\phi_{t'\tau} + 0.012A_{t'_0}M_{10}cos\phi_{t'1} \\
+& \; 0.078A_{t'_0}M_{20}cos\phi_{t'2} + 0.35A_{t'_0}M_{30}cos\phi_{t'3} - 0.036M^2_{10} \\
-& \; 0.36M^2_{20} + 0.68m^2_{30} - 0.015\mu'_0A_{b_0}cos\phi_{4b} \\
-& \; 0.042\mu'_0A_{t_0}cos\phi_{4t} - 0.017\mu'_0A_{\tau_0}cos\phi_{4\tau} + 0.0065\mu'_0M_{10}cos\phi_{41} \\
+& \; 0.037\mu'_0M_{20}cos\phi_{42} + 0.14\mu'_0M_{30}cos\phi_{43}. \qquad (88)
\end{aligned}
$$

B.3 trilinear terms in the NHSSM

At the low values of $tan\beta$:

$$
\begin{aligned}
A_t(t_Z) &= -0.00063A_{b_0} + 0.22A_{t_0} + 3.6\times 10^{-7}A_{\tau_0} - 0.029M_{10} - 0.23M_{20} - 1.9M_{30} \\
A_b(t_Z) &= 0.99A_{b_0} - 0.13A_{t_0} - 0.00079A_{\tau_0} - 0.033M_{10} - 0.48M_{20} - 3M_{30} \\
A_\tau(t_Z) &= -0.0032A_{b_0} + 0.00029A_{t_0} + A_{\tau_0} - 0.16M_{10} - 0.53M_{20} + 0.005M_{30} \\
A_{t'}(t_Z) &= 0.00061A_{b'_0} - 2.8\times 10^{-7}A_{\tau'_0} + 0.49A_{t'_0} + 0.46\mu'_0 \\
A_{b'}(t_Z) &= 0.63A_{b'_0} - 0.00044A_{\tau'_0} + 0.14A_{t'_0} + 0.19\mu'_0 \\
A_{\tau'}(t_Z) &= -0.0018A_{b'_0} + 0.49A_{\tau'_0} - 0.00026A_{t'_0} + 0.47\mu'_0. \qquad (89)
\end{aligned}
$$

When $tan\beta$ is high:

$$\begin{aligned}
A_t(t_Z) &= -0.05A_{b_0} + 0.21A_{t_0} + 0.0045A_{\tau_0} - 0.027M_{10} - 0.21M_{20} - 1.8M_{30} \\
A_b(t_Z) &= 0.38A_{b_0} - 0.072A_{t_0} - 0.055A_{\tau_0} - 0.0092M_{10} - 0.25M_{20} - 2.1M_{30} \\
A_\tau(t_Z) &= -0.26A_{b_0} + 0.027A_{t_0} + 0.62A_{\tau_0} - 0.11M_{10} - 0.32M_{20} + 0.44M_{30} \\
A_{t'}(t_Z) &= 0.082A_{b'_0} - 0.0065A_{\tau'_0} + 0.42A_{t'_0} + 0.18\mu'_0 \\
A_{b'}(t_Z) &= 0.54A_{b'_0} - 0.069A_{\tau'_0} + 0.12A_{t'_0} + 0.083\mu'_0 \\
A_{\tau'}(t_Z) &= -0.29A_{b'_0} + 0.6A_{\tau'_0} - 0.036A_{t'_0} + 0.4\mu'_0. \qquad (90)
\end{aligned}$$

Note that for the same values of $tan\beta$ one-loop MSSM results can be obtained from the NHSSM solutions via the appropriate transformations (see text for details).

C MSSM & NHSSM under universality assumption

For the sake of simplicity and completeness, we also provide the solutions using (4), both in the MSSM and NHSSM; $mass^2$ and trilinear terms are presented in the following subsections.

C.1 MSSM under universal terms

With the help of (4) for low $tan\beta$ MSSM results are

$$\begin{aligned}
m^2_{H_u}(t_Z) &= -0.087A_0^2 + 0.38A_0M - 2.8M^2 - 0.16m_0^2, \\
m^2_{H_d}(t_Z) &= -0.0033A_0^2 + 0.011A_0M + 0.49M^2 + 0.99m_0^2, \\
m^2_{t_L}(t_Z) &= -0.03A_0^2 + 0.13A_0M + 5.7M^2 + 0.61m_0^2, \\
m^2_{t_R}(t_Z) &= -0.058A_0^2 + 0.25A_0M + 4.1M^2 + 0.22m_0^2, \\
m^2_{b_R}(t_Z) &= -0.0017A_0^2 + 0.0072A_0M + 6.3M^2 + 0.99m_0^2, \\
m^2_{l_L}(t_Z) &= -0.00078A_0^2 + 0.00067A_0M + 0.52M^2 + m_0^2, \\
m^2_{l_R}(t_Z) &= -0.0016A_0^2 + 0.0013A_0M + 0.15M^2 + m_0^2, \\
m^2_3(t_Z) &= -0.38A_0\mu_0 + 0.96m^2_{30} + 0.26M\mu_0, \\
A_t(t_Z) &= 0.22A_0 - 2.2M, \\
A_b(t_Z) &= 0.074A_0 - 0.3M, \\
A_\tau(t_Z) &= 0.052A_0 - 0.036M, \qquad (91)
\end{aligned}$$

for high $tan\beta$ MSSM results can be written as

$$\begin{aligned}
m^2_{H_u}(t_Z) &= -0.061A_0^2 + 0.27A_0M - 2.6M^2 - 0.12m_0^2, \\
m^2_{H_d}(t_Z) &= -0.1A_0^2 + 0.32A_0M - 2.M^2 - 0.066m_0^2, \\
m^2_{t_L}(t_Z) &= -0.041A_0^2 + 0.19A_0M + 4.9M^2 + 0.36m_0^2, \\
m^2_{t_R}(t_Z) &= -0.041A_0^2 + 0.18A_0M + 4.3M^2 + 0.25m_0^2, \\
m^2_{b_R}(t_Z) &= -0.042A_0^2 + 0.21A_0M + 4.7M^2 + 0.46m_0^2,
\end{aligned}$$

$$
\begin{aligned}
m^2_{l_L}(t_Z) &= -0.037A_0^2 + 0.0099A_0M + 0.51M^2 + 0.75m_0^2,\\
m^2_{l_R}(t_Z) &= -0.075A_0^2 + 0.02A_0M + 0.12M^2 + 0.49m_0^2,\\
m^2_3(t_Z) &= -0.5A_0\mu_0 + 0.68m^2_{30} + 0.59M\mu_0,\\
A_t(t_Z) &= 0.16A_0 - 2M,\\
A_b(t_Z) &= 0.21A_0 - 2M,\\
A_\tau(t_Z) &= 0.2A_0 + 0.0041M.
\end{aligned} \tag{92}
$$

C.2 NHSSM under universal terms

with the help of (4) again for low $tan\beta$ $mass^2$ terms:

$$
\begin{aligned}
m^2_{H_u}(t_Z) &= -0.087A_0^2 + 0.10A_0'^2 - 0.16m_0^2 - 2.84M^2 + 0.067A_0'\mu_0' + 0.14\mu_0'^2 + 0.38A_0M,\\
m^2_{H_d}(t_Z) &= -0.0033A_0^2 - 0.37A_0'^2 + 0.99m_0^2 + 0.49M^2 - 0.31A_0'\mu_0' + 0.6\mu_0'^2 + 0.011A_0M,\\
m^2_{t_L}(t_Z) &= -0.03A_0^2 - 0.089A_0'^2 + 0.61m_0^2 + 5.7M^2 - 0.08A_0'\mu_0' + 0.3\mu_0'^2 + 0.13A_0M,\\
m^2_{t_R}(t_Z) &= -0.058A_0^2 - 0.18A_0'^2 + 0.22m_0^2 + 4.1M^2 - 0.16A_0'\mu_0' + 0.6\mu_0'^2 + 0.25A_0M,\\
m^2_{b_R}(t_Z) &= -0.0017A_0^2 - 0.00079A_0'^2 + 0.99m_0^2 + 6.3M^2 - 0.00015A_0'\mu_0' + 0.0029\mu_0'^2\\
&+ 0.0072A_0M,\\
m^2_{l_L}(t_Z) &= -0.00078A_0^2 - 0.00023A_0'^2 + m_0^2 + 0.52M^2 - 0.00022A_0'\mu_0' + 0.0012\mu_0'^2\\
&+ 0.00067A_0M,\\
m^2_{l_R}(t_Z) &= -0.0016A_0^2 - 0.00045A_0'^2 + m_0^2 + 0.15M^2 - 0.00044A_0'\mu_0' + 0.0025\mu_0'^2\\
&+ 0.0013A_0M,\\
m^2_3(t_Z) &= -0.27A_0A_0' - 0.56M^2 + 0.96m^2_{30} - 0.099A_0\mu_0' + 0.5A_0'M + 0.32\mu_0'M,\\
A_t(t_Z) &= 0.22A_0 - 2.2M,\\
A_b(t_Z) &= 0.86A_0 - 3.6M,\\
A_\tau(t_Z) &= 0.99A_0 - 0.68M,\\
A_{t'}(t_Z) &= 0.49A_0' + 0.46\mu_0',\\
A_{b'}(t_Z) &= 0.77A_0' + 0.19\mu_0',\\
A_{\tau'}(t_Z) &= 0.49A_0' + 0.47\mu_0'.
\end{aligned} \tag{93}
$$

For high $tan\beta$:

$$
\begin{aligned}
m^2_{H_u}(t_Z) &= -0.061A_0^2 - 0.12A_0'^2 - 0.12m_0^2 - 2.6M^2 - 0.036A_0'\mu_0' - 0.0083\mu_0'^2 + 0.27A_0M,\\
m^2_{H_d}(t_Z) &= -0.1A_0^2 - 0.21A_0'^2 - 0.066m_0^2 - 2.M^2 - 0.11A_0'\mu_0' + 0.13\mu_0'^2 + 0.32A_0M,\\
m^2_{t_L}(t_Z) &= -0.041A_0^2 - 0.1A_0'^2 + 0.36m_0^2 + 4.9M^2 - 0.04A_0'\mu_0' + 0.35\mu_0'^2 + 0.19A_0M,\\
m^2_{t_R}(t_Z) &= -0.041A_0^2 - 0.11A_0'^2 + 0.25m_0^2 + 4.3M^2 - 0.065A_0'\mu_0' + 0.43\mu_0'^2 + 0.18A_0M,\\
m^2_{b_R}(t_Z) &= -0.042A_0^2 - 0.086A_0'^2 + 0.46m_0^2 + 4.7M^2 - 0.014A_0'\mu_0' + 0.28\mu_0'^2 + 0.21A_0M,\\
m^2_{l_L}(t_Z) &= -0.037A_0^2 - 0.027A_0'^2 + 0.75m_0^2 + 0.51M^2 - 0.023A_0'\mu_0' + 0.14\mu_0'^2\\
&+ 0.0099A_0M,
\end{aligned}
$$

$$
\begin{aligned}
m^2_{l_R}(t_Z) &= -0.075A_0^2 - 0.054A_0'^2 + 0.49m_0^2 + 0.11M^2 - 0.047A_0'\mu_0' + 0.27\mu_0'^2 + 0.02A_0M, \\
m^2_3(t_Z) &= -0.23A_0A_0' - 0.4M^2 + 0.68m^2_{30} - 0.074A_0\mu_0' + 0.8A_0'M + 0.19\mu_0'M, \\
A_t(t_Z) &= 0.16A_0 - 2.1M, \\
A_b(t_Z) &= 0.25A_0 - 2.3M \\
A_\tau(t_Z) &= 0.39A_0 + 0.0081M, \\
A_{t'}(t_Z) &= 0.5A_0' + 0.18\mu_0', \\
A_{b'}(t_Z) &= 0.6A_0' + 0.083\mu_0', \\
A_{\tau'}(t_Z) &= 0.28A_0' + 0.4\mu_0'.
\end{aligned} \tag{94}
$$

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